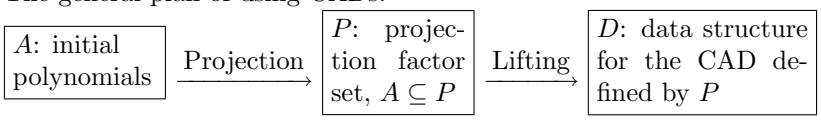
ISSAC 2004 Tutorial *Cylindrical Algebraic Decomposition*, Christopher W. Brown

1. Introduction
   (a) Preface: Overarching goal is to compute symbolically with semi-algebraic sets in practice. Define semi-algebraic. Define Tarski formula.
   (b) Introduction proper
   (c) Three motivating examples: An epidemiological model and criteria for the existence of an "endemic equilibrium" (quantifier elimination), triangles and the existence of "the external trisector of B w.r.t. A" (formula simplification), Rolle's theorem and the existence of unrealizable "legal sign stack sequences" (using properties of CADs in new ways).
2. An informal introduction to CAD (Getting the intuition)
   (a) natural algebraic decomposition, the level of a polynomial
   (b) CADs as mathematical objects: induced CADs, projection factor sets
   (c) CADs as data structures: sample points, lifting
   (d) Representing sets as CADs, quantifier elimination (Q.E.), formula simplification
   (e) The general plan of using CADs:



3. CAD more formally
   (a) This section covers: Projection, stack construction, solution formula construction. Does not cover: propagation, simplification, adjacency computation, etc.
   (b) Define cylindrical, cylindrical algebraic decompostion, projection factor set.
   (c) How do you make a decomposition cylindrical? Decompose the space of one dimension lower so that regions are cylindrically arranged over these new base regions.
   (d) Deliniability. Connection between delineability of polynomials and cylindricity of the natural algebraic decomposition they define. Delineability and our Rolle's Theorem/Legal sign stack sequence problem.
   (e) Projection Operators, the Brown-McCallum projection operator, examples.
   (f) Lifting, examples, issues.
   (g) Solution formula construction, "simple" formulas, projection definability, coping with projection undefinability.
4. Using CAD effectively
   (a) Limitations, both theoretical and practical.
   (b) Implementations
   (c) Solving problems efficiently with CAD
      i. Effects of variable orderings.
      ii. Preparing input: break into pieces, perform trivial eliminations, etc.
      iii. Partial CAD
      iv. Important special case of partial CAD: full dimensional cells only!
      v. Go beyond Q.E. and simplification. Make use of the structure of CADs!
5. Case studies