This is a mult-section quiz. You may not communicate to anyone about it until I notify you that you can.

SI340 Quiz 7 Name _____ Alpha _____

1. Using the Pumping Lemma, prove the language L = {w in {a,b}* | $N_a(w) < N_b(w)$} is not regular.
   Assume L is regular.
   Let x = $a^N b^{N+1}$
   |x| = | $a^N b^{N+1}$ | = 2n+1 > N
   Let u = a^i where i+j <= N and j>0
   v = a^j
   w = $a^{N-i-j} b^{N+1}$
   Let m = 2 so $uv^2w = a^i a^j a^j a^{N-i-j} b^{N+1} = a^{N+j} b^{N+1}$ which is not in L because since j>0 there are at least as many if not more a's than b's in the string. This contradicts the pumping lemma and the assumption that L is regular is false. Hence L is not regular.

2. Using Myhill-Nerode, prove the language L = {ww | w in$\{a,b\}^*$} is not regular.

   Let S = {a^N | N > 0} S = {a, aa, aaa, aaaa, ….} and is therefore infinite.

   We need an x and y in S so let
   x = a^i
   y = a^j where i not equal to j

   Let z = $b^i a^i b^i$.

   Now xz = $a^i b^i a^i b^i$ which is in L.
   yz = $a^j b^i a^i b^i$ which is not in L since there are more or fewer a's in the first part of the string than in the second part of the string.

   Hence the two strings x and y are distinguishable with respect to L and L is not regular.