This is a mult-section quiz. You may not communicate to anyone about it until I notify you that you can.

SI340 Quiz 11 Name _____ Alpha _____

1. Using the Pumping Lemma for Context Free Languages, prove that the language $L=\{a^N b^M c^{NM} \mid N, M > 0\}$ is not context free. Reminder: NM means N multiplied by M.

Assume that L is a context free language.

Let $w = a^N b^M c^{NM}$ Then $| a^N b^M c^{NM} | = N + M + NM >= N$

Case 1: vxy contains all a's

Let $u = a^i$

$v = a^j$

$x = a^k$

$y = a^p$

$z = a^{N-i-j-k-p} b^M c^{NM}$ where $j+k+p <=N$ and $j+p >0$.

Let m = 2. $vv^2xy^2z = a^i a^j a^j a^k a^p a^p a^{N-i-j-k-p} b^M c^{NM} = a^{N+j+p} b^M c^{NM}$ which is not in L. Since $j+p >0$ the number of a's is at least N+1. In order for it to be in the language the number of c's would have to be (N+1)M = NM + M. But the number of c's is NM and hence the string is not in L.

Case 2: vxy contains all b's After pumping, the string vv^2xy^2z would be at least $a^N b^{M+1} c^{NM}$ which is not in L because N(M+1) does not equal NM.

Case 3: vxy contains all c's. After pumping, the string vv^2xy^2z would be at least $a^N b^M c^{NM+1}$ which is not in L because NM does not equal NM+1.

Case 4: vxy crosses the a-b boundary. After pumping, the string vv^2xy^2z would be at least $a^N b^{M+1} c^{NM}$ or $a^{N+1} b^M c^{NM}$ neither of which is in L because NM does not equal N(M+1) or M(N+1).

Case 5: vxy crosses the b-c boundary After pumping, the string vv^2xy^2z would be at least $a^N b^{M+1} c^{NM}$ or $a^N b^M c^{NM+1}$ neither of which is in L because NM does not equal N(M+1) or NM+1.

Because in all cases the pumped string produces a string that is not in L, this contradicts the pumping lemma which means the assumption is wrong and L is not context free.