NAME: SOLUTIONS **ALPHA NUMBER:** ______________
INSTRUCTOR: ______________________________ **SECTION:** ______________

0755-1055 Friday 7 December 2007 SHOW ALL WORK IN THIS TEST PACKAGE

Tear off the last sheet of this exam. It contains problems 27, 28, 29, 30. Do these problems first WITHOUT YOUR CALCULATOR. When you are finished with these 4 problems, return the sheet to your instructor, take out you calculator, and complete the rest of the exam.

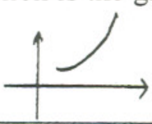
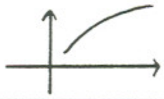
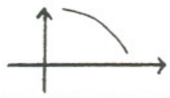
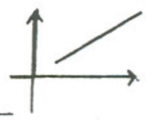
PART ONE: MULTIPLE CHOICE (50%). The first 20 problems are multiple-choice. Fill in the best answer on your Scantron bubble sheet. Write your name, alpha number, instructor and section on this test and your bubble sheet and bubble in your alpha number. There is no extra penalty for wrong answers on the multiple choice. Show all your scratch work on this test.
CALCULATORS PERMITTED FOR THIS SECTION.

1. For the right triangle shown, which trigonometric function applied to θ gives a result of 3 ?

$c^2 = a^2 + b^2$
$10 = 9 + b^2 \Rightarrow b = 1$
$\cot(\theta) = 3/1$

a) sin b) cos c) tan d) cot e) sec

2. Which is the graph of a function f with $f' > 0$ and $f'' < 0$?

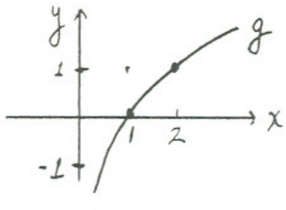
a)  b)  c)  d)  e) 

3. Solve for x to two decimal places if $7 = 3 + 4(e^{3x} - 1)$.

a) -2.00 b) 0.00
c) 0.23 d) 1.50
e) 2.72

$4 = 4(e^{3x} - 1)$
$1 = e^{3x} - 1$
$e^{3x} = 2$
$3x = \ln(2)$
$x = \ln(2)/3 \doteq 0.23$

4. If $f(x) = \begin{cases} x-1, & x \le 3 \\ x^2 + \dfrac{1}{x}, & 3 < x \end{cases}$ and g^{-1} is the inverse for the function g graphed on the right, find $f(g^{-1}(1))$.



$= f(2) = 2 - 1 = 1$

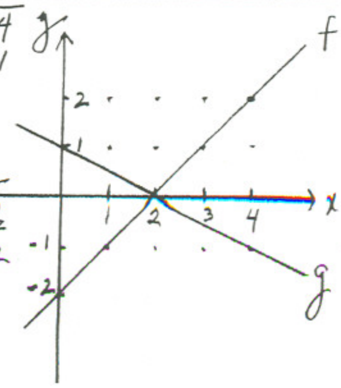
a) 0 b) 2 c) -1 d) 1 e) 4.5

5. Use the graph on the right to find $\lim_{x \to 4} f(x)g(x)/\sqrt{x}$. $= (2)(-1)/\sqrt{4} = -2/2 = -1$

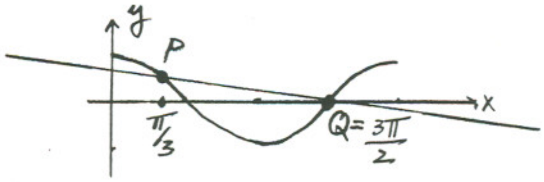
a) 2 b) -1 c) -2 d) - 6 e) does not exist

6. Use the same graph on the right to find $\lim_{x \to 2} \dfrac{f(x)}{g(x)} \overset{\mathcal{L}'}{=} \lim_{x \to 2} \dfrac{f'(x)}{g'(x)} = \dfrac{1}{-1/2} = -2$

a) -1/2 b) 0 c) -2 d) 1 e) does not exist



7. The graph for $y=\cos(x)$ is shown on the right. Find the slope of the line through points P and Q. (P has an x coordinate of $\pi/3$ radians and Q is on the x-axis.)



a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{-3}{7\pi}$

d) $\frac{3\sqrt{3}}{8\pi}$ e) $\frac{\sqrt{3}}{\pi}$

$m=\frac{\Delta y}{\Delta x}=\frac{0-\cos(\frac{\pi}{3})}{\frac{3\pi}{2}-\frac{\pi}{3}}=\frac{-\frac{1}{2}}{\frac{9\pi-2\pi}{6}}=-\frac{1}{2}\Big/\frac{7\pi}{6}=-3/7\pi$

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8. Which of the following statements implies that a function f has a horizontal asymptote ?

a) $\lim_{x\to\infty} f(x)=\infty$ b) $\lim_{x\to\infty} f(x)=2$ c) $\lim_{x\to 4} f(x)=0$ d) $\lim_{x\to 4} f(x)=-\infty$ e) $\lim_{x\to 0} f(x)=0$

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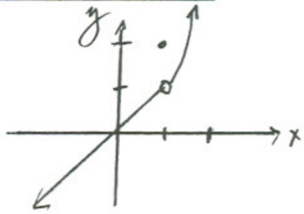
9. If $f(x)=\begin{cases} x, & x<1 \\ 2, & x=1 \\ x^2, & 1<x \end{cases}$, then $\lim_{x\to 1} f(x)$ is which of the following?



a) *does not exist* b) 0 c) 1 d) 2 e) 3

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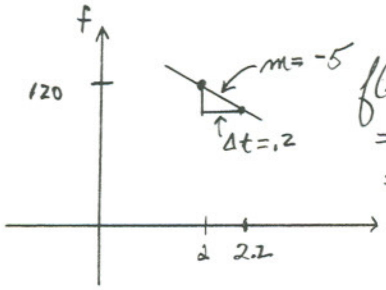
10. Let $f(t)$ measure the temperature (^oF) of a cup of coffee at time t (min). If $f(2)=120\ ^oF$ and $f'(2)=-5\ ^oF/\text{min}$, a linear approximation of the temperature of the coffee at time 2.2 min is:



$f(2.2)\doteq f(2)+f'(2)\Delta t$
$=120+(-5)(.2)$
$=119\ ^oF$

a) 100^oF b) 115^oF c) 125^oF
d) 121^oF e) 119^oF

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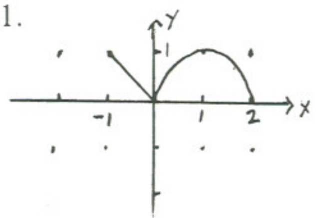
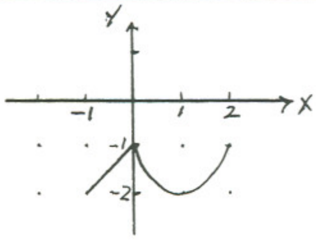
11. 

If f is the function graphed on the left, then the function graphed on the right could be:



a) $-f(x)-1$ b) $-2f(x-1)$ c) $\frac{1}{f(x)}$ d) $-2f(x)$ e) $f^{-1}(x+1)$

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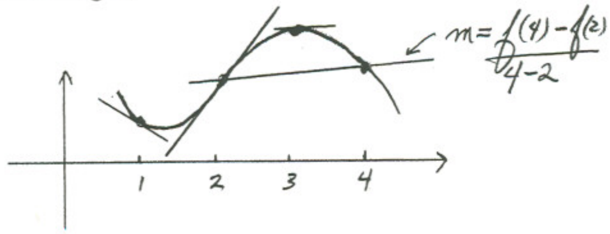
12. For the function f whose graph is shown on the right, Which of the following is largest?

a) $f'(1)$ b) $f'(2)$ c) $f'(3)$

d) $f''(4)$ e) $\underbrace{\frac{f(4)-f(2)}{2}}$

<0

slope of secant line



13. Use your calculator to sketch the graphs for $y = x^5 + x^3 + 2$ and $y = \cos(x^2) + x$ and determine the point of intersection accurate to one decimal place.

a) (0 , 0) b) (-2.1 , 0) c) (3.3 , .4) d) (-1.1 , -.7) e) d.n.e.

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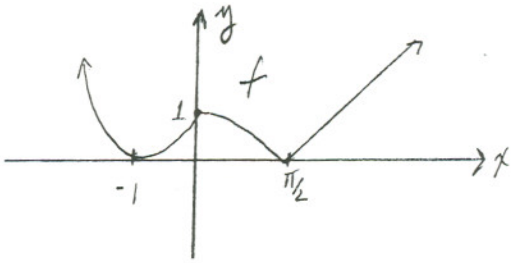
14. At what value of x is the function f discontinuous if

$$f(x) = \begin{cases} (x+1)^2, & x \le 0 \\ \cos(x), & 0 < x < \pi/2 \\ x - \pi/2, & \pi/2 \le x \end{cases} \quad ?$$



a) - 1 b) 0 c) 1
d) $\pi/2$ e) none

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15. If the table on the right gives the velocity of a car at various times, find the average acceleration of the car over the time interval [1, 3].

t (sec)	0	1	2	3	4
v (ft/s)	0	9	20	33	52

ave accel $= \dfrac{v(3)-v(1)}{3-1} = \dfrac{33-9}{2} = 12$

a) 21 b) 12 c) 20 d) 11 e) 33

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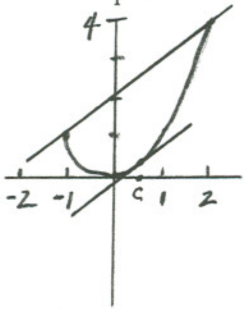
16. Find the number c predicted by the Mean Value Theorem for $f(x) = x^2$ on [-1 , 2].



a) -1
b) 1/2
c) 1
d) 2
e) none

$f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$2c = \dfrac{4-1}{2-(-1)} = \dfrac{3}{3} = 1$

$c = 1/2$

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17. Use Newton's Method with initial approximation $x_1 = 2$ to find x_2, the second approximation to a root for the equation $x^2 - 1 = 0$.

a) 1.00
b) 1.25
c) -2
d) 2
e) none

$f(x) = x^2 - 1$
$f'(x) = 2x$

$x_2 = x_1 - f(x_1)/f'(x_1)$
$= 2 - f(2)/f'(2)$
$= 2 - 3/4$
$= 1.25$

18. Air is pumped into a spherical balloon ($V = \frac{4}{3}\pi r^3$). Find the rate of change of the volume with respect to time at the moment when the radius is $2\,m$ and increasing at $0.1\,m/s$.

a) $0.1\ m^3/s$
b) $1.6\pi\ m^3/s$
c) $16\pi\ m^3/s$
d) $16\pi\ m/s$
e) $4\pi\ m^2/s$

$$V(t) = \frac{4}{3}\pi(r(t))^3$$
$$\frac{dV(t)}{dt} = 4\pi\,(r(t))^2\frac{dr(t)}{dt}$$
$$= 4\pi(2m)^2(.1\,m/s)$$
$$= 1.6\pi\,m^3/s$$

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19. Find an equation for the line tangent to the curve given implicitly by $xy + y^2 = -2$ at the point (3 , -1).

a) $y = x$
b) $y = -x$
c) $y = x - 4$
d) $y = x + 4$
e) none of the above

tangent line
$$y - y_0 = m(x - x_0)$$
$$y - (-1) = m(x-3)$$
$$m = \frac{dy}{dx}\Big|_{(3,-1)}$$
$$\frac{d}{dx}[xy + y^2] = \frac{d}{dx}[-2]$$
$$1y + xy' + 2yy' = 0$$
$$y'[x+2y] = -y$$
$$y' = \frac{-y}{x+2y}\Big|_{(3,-1)}$$
$$= \frac{1}{3-2} = 1$$
$$\Rightarrow y + 1 = 1(x-3)$$
$$y = x - 4$$

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20. Find $F(x)$, the antiderivative of $f(x) = 6x^2 - 2\sin(x)$, if $F(0) = 4$.

a) $F(x) = 12x - 2\cos(x) + 6$
b) $F(x) = 12x - 2\cos(x) + 4$
c) $F(x) = 2x^3 + 2\cos(x) + 2$
d) $F(x) = 2x^3 + 2\cos(x) + 4$
e) none of the above

$$F(x) = \frac{6x^3}{3} + 2\cos(x) + C$$
$$F(0) = 4 = 0 + 2(\cos(0)) + C$$
$$4 = 2 + C \Rightarrow C = 2$$
$$F(x) = 2x^3 + 2\cos(x) + 2$$

Part Two. Longer Answers (50%). These are not multiple choice. Again, SHOW ALL YOUR WORK ON THESE TEST PAGES. **CALCULATORS PERMITTED FOR ALL PROBLEMS <u>EXCEPT</u> 27, 28, 29, 30.**

21. The graph of a function f is shown on the right.

a) Explain why f has an inverse function f^{-1}.

Because f is 1-1 (satisfies the horizontal line test).

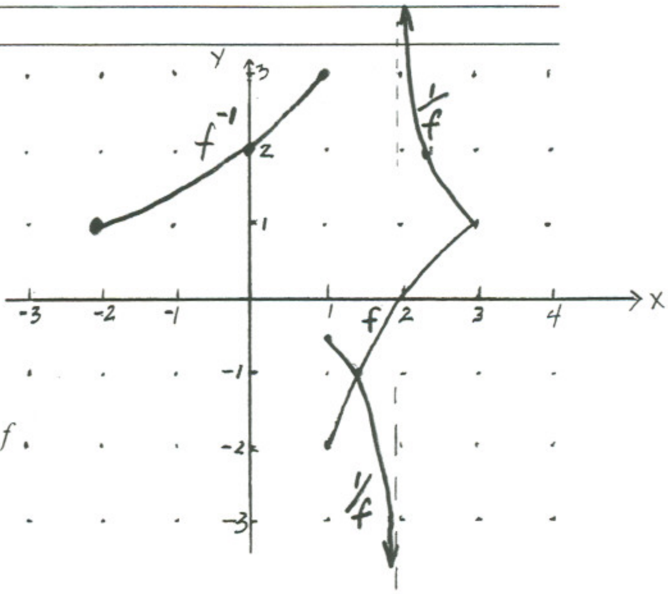
b) What is the domain and range of f^{-1}?

domain of $f^{-1} = [-2, 1]$

range of $f^{-1} = [1, 3]$

c) Sketch the graph of f^{-1} on the same axes with f.

d) Sketch the graph of $1/f$ (the reciprocal of f) on the same axes with f.

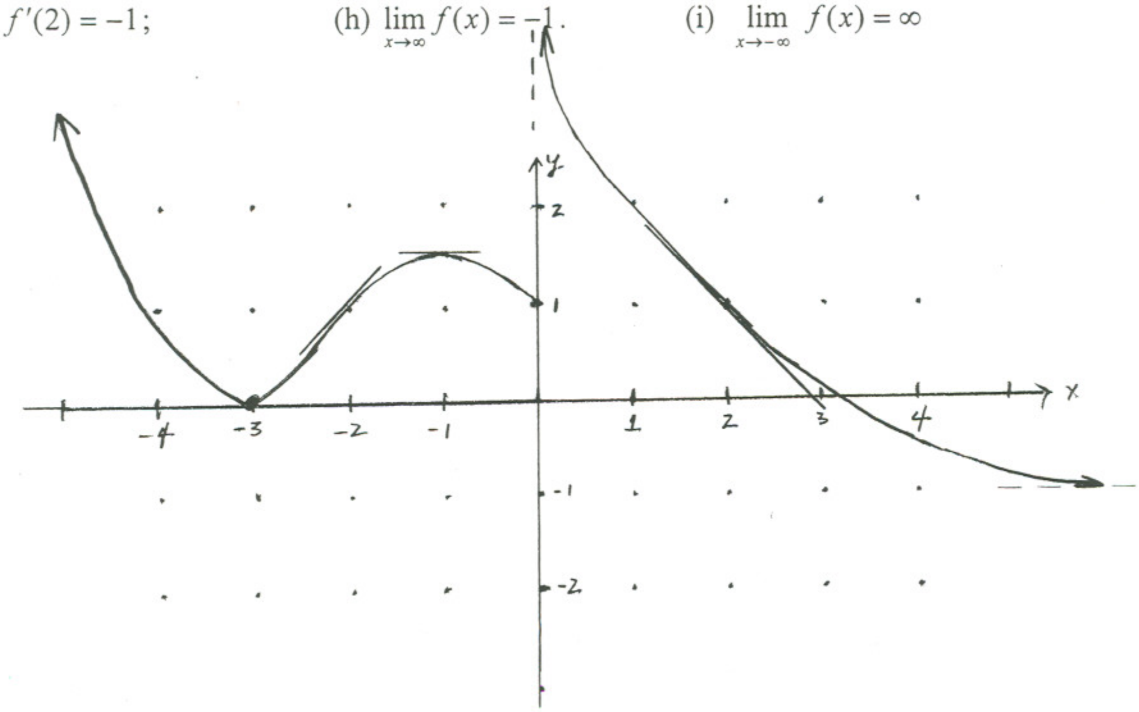


22. Sketch the graph <u>of a single function</u> (on the axes below) which satisfies all of the following:

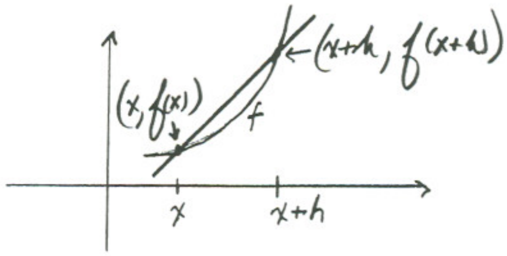
(a) $\lim_{x\to -3} f(x) = 0$; (b) $f'(-2) = 1$; (c) $f'(-1) = 0$;

(d) $\lim_{x\to 0^-} f(x) = 1$; (e) $\lim_{x\to 0^+} f(x) = \infty$; (f) $\lim_{x\to 2} f(x) = 1$;

(g) $f'(2) = -1$; (h) $\lim_{x\to\infty} f(x) = -1$. (i) $\lim_{x\to -\infty} f(x) = \infty$



23. a) Find an expression for the slope of the secant line for the function f drawn on the right.



$$m_{sec} = \frac{f(x+h) - f(x)}{h}$$

b) Write the limit definition for $f'(x)$ and use it to find $f'(x)$ for $f(x) = \dfrac{1}{x}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{\left(\frac{\cancel{x} - (\cancel{x}+h)}{x(x+h)}\right)}{h}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{x(x+h) \cdot \cancel{h}} = \lim_{h \to 0} \frac{-1}{x(x+h)} = \frac{-1}{x^2}$$

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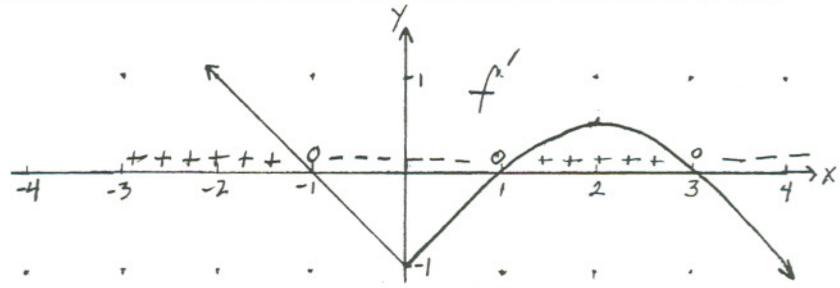
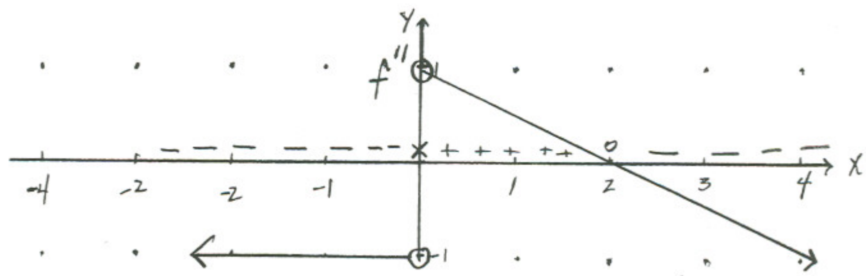
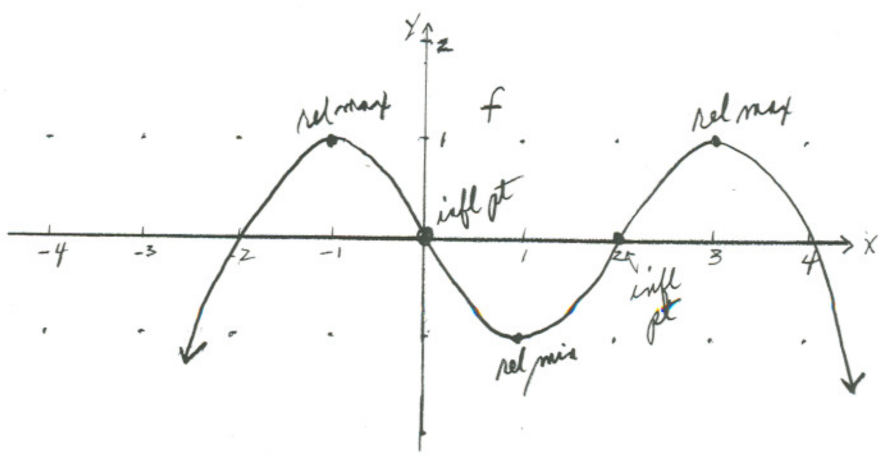
24. The graph of the derivative f' is shown.

a) Sketch the graph of f'' on the axes provided.

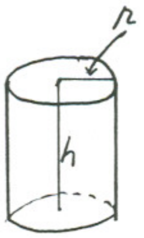
b) Use information from the graphs of f' and f'' (i.e. their signs) to sketch a possible graph for a continuous function f. Assume that $f(0) = 0$. Label all relative maximums, minimums, and inflection points.







25) Assume that the trunk of a tree can be modeled using a right circular cylinder. What is the rate of change with respect to time of the volume of the tree at the moment when the radius is 3 ft and increasing at a rate of 0.2 ft/year and the height is 15 ft and increasing at a rate of 2.0 ft/year. (Recall that for a cylinder, $V(t)=\pi(r(t))^2\cdot(h(t))$).



$$V(t)=\pi\left(r(t)\right)^2\cdot h(t)$$

$$\frac{dV}{dt}(t)=\pi\left[2r(t)r'(t)\cdot h(t)+\left(r(t)\right)^2\cdot h'(t)\right]$$

$$=\pi\left[2(3\,ft)\left(.2\,\frac{ft}{yr}\right)(15\,ft)+(3\,ft)^2\left(2\,\frac{ft}{yr}\right)\right]$$

$$=\pi\left[18\,\frac{ft^3}{yr}+18\,\frac{ft^3}{yr}\right]$$

$$=36\pi\ ft^3/yr$$

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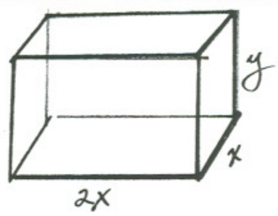
26) An aquarium is to be built in the shape of a rectangular box with no top. The length of the aquarium must to be twice the width and the surface area must be 36 ft^2 of material. What is the maximum volume that the aquarium can hold?



maximize $V=(2x)(x)(y)=2x^2y$

$$\underbrace{2x^2}_{\text{bottom}}+\underbrace{2(xy)}_{\text{left + right}}+\underbrace{2(2xy)}_{\text{front + back}}=36$$

$$2x^2+6xy=36\Rightarrow y=\frac{36-2x^2}{6x}=\frac{6}{x}-\frac{x}{3}$$

$$\Rightarrow V=2x^2\left(\frac{6}{x}-\frac{x}{3}\right)=12x-\frac{2}{3}x^3;\quad x\in(0,\sqrt{18})$$

$$V'=12-2x^2=0\Rightarrow x=+\sqrt{6},\ \cancel{-\sqrt{6}}$$

$$V''=-4x\Big|_{x=\sqrt{6}}<0\Rightarrow x=\sqrt{6}\text{ is the maximum}$$

$$\Rightarrow V=12x-\frac{2}{3}x^3\Big|_{x=\sqrt{6}}=12\sqrt{6}-\frac{2}{3}6\sqrt{6}=8\sqrt{6}\ ft^3$$

PAGE 8 IS BLANK AND CAN BE USED AS SCRATCH PAPER.

CALCULATORS NOT PERMITTED FOR 27, 28, 29, 30.

27. Find $\frac{dy}{dx}$ for the following functions. (Do not simplify your answers.)

(a) $y=(x+3)^5\sin(4x)$; $y' = 5(x+3)^4 \sin(4x) + (x+3)^5 \cos(4x)\cdot 4$

(b) $y=\frac{\sqrt[3]{x}}{\tan(x\ )}$; $y' = \dfrac{\frac{1}{3}x^{-2/3}\tan(x) - x^{1/3}\sec^2(x)}{\tan^2(x)}$

(c) $y=\arctan(\sqrt{x}))$; $y' = \dfrac{1}{1+(\sqrt{x})^2}\cdot \frac{1}{2}x^{-1/2}$

(d) $y=x^{x^3}$; $\ln(y) = \ln x^{x^3} = x^3\ln(x) \Rightarrow \frac{1}{y}y' = [3x^2\ln(x) + x^3\cdot\frac{1}{x}]$
$\Rightarrow y' = x^{x^3}[3x^2\ln(x) + x^2]$

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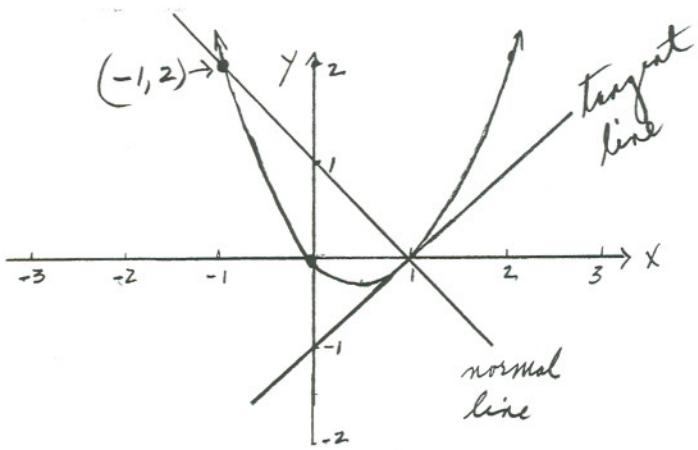
28. (a) Find an equation for the line tangent to the parabola $y=x^2-x$ at the point (1 , 0).

$y - 0 = m(x-1)$
$m = y'|_{x=1} = 2x-1|_{x=1} = 1$
$\Rightarrow y = 1(x-1) = x-1$



(b) Sketch a graph of the parabola and its tangent line from (a) on the given axes.

$y = x^2 - x = x(x-1)$
$\Rightarrow$ roots at $x=0,\ x=1$.

(c) Find an equation for, and sketch (on the axes above) the normal (perpendicular) line to the parabola at the point (1 , 0).

$y - 0 = m_\perp(x-1) = \frac{-1}{1}(x-1)$
$\Rightarrow y = -x+1$

(d) Find any other points where the normal line intersects the parabola

$x^2 - x = -x+1$
$x^2 = 1 \Rightarrow x = -1, \cancel{1} \Rightarrow$ intersection pt $= (-1,2)$

29. Use f' and f'' to graph $f(x) = x^3 - 3x^2$. Label all relative maximums and minimums and inflection points.

$f(x) = x^3 - 3x^2 = x^2(x-3)$

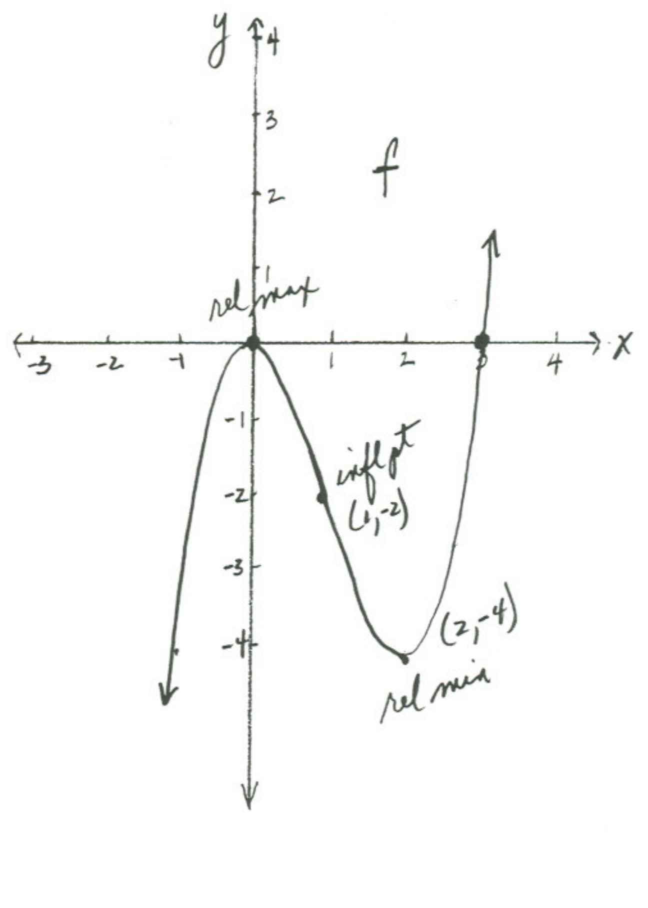
$f'(x) = 3x^2 - 6x = 3x(x-2)$

$f''(x) = 6x - 6 = 6(x-1)$

sign of f: $- - -\ 0\ - - -\ 0\ + + +$ (at $-1\ 0\ 1\ 2\ 3\ 4$)

sign of f': $+ + + +\ 0\ - - -\ 0\ + + +$ (at $-1\ 0\ 1\ 2\ 3\ 4$)

sign of f'': $- - -\ 0\ + + + +$ (at $-1\ 0\ 1\ 2\ 3\ 4$)



30. Find the following limits. Show all work.

(a) $\lim\limits_{x\to 0} \frac{\sin(2x)}{x} \overset{\mathcal{L}'}{=} \lim\limits_{x\to 0} \frac{(\sin(2x))'}{x'} = \lim\limits_{x\to 0} \frac{2\cos(2x)}{1} = 2$

(b) $\lim\limits_{x\to 0} \frac{\sin(2x)}{x+1} = \frac{0}{1} = 0$

(c) $\lim\limits_{x\to 0^+} x\ln(x) = \lim\limits_{x\to 0^+} \frac{\ln(x)}{1/x} \overset{\mathcal{L}'}{=} \lim\limits_{x\to 0^+} \frac{1/x}{-1/x^2} \overset{alg}{=} \lim\limits_{x\to 0^+} \left(\frac{1}{x}\right)\left(\frac{-x^2}{1}\right) = \lim\limits_{x\to 0^+} (-x) = 0$

(d) $\lim\limits_{x\to 1} x\ln(x) = (1)(0) = 0$