

Calculus I Chapter 4.5 Practice Problem Solutions

1. $f(x) = x^2 + \frac{1}{x}$ is continuous on $[.5, 2]$
 $f'(x) = 2x - \frac{1}{x^2} = 0 \Rightarrow 2x^3 = 1$
 $\Rightarrow x = \sqrt[3]{\frac{1}{2}} = \frac{1}{\sqrt[3]{2}}$ (only crit pt)

x	f(x)
.5	$\frac{1}{4} + 2 = 2.25$
2	$4 + \frac{1}{2} = 4.5$
$\sqrt[3]{\frac{1}{2}}$	$(\frac{1}{\sqrt[3]{2}})^2 + \sqrt[3]{\frac{1}{2}} \approx 1.89$

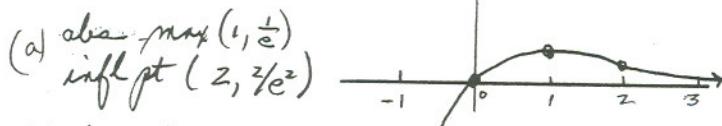
\Rightarrow abs min at $\sqrt[3]{\frac{1}{2}}$, abs max at 2.

2. $f(x) = x e^x$; $f'(x) = (1-x)e^{-x}$; $f''(x) = (x-2)e^{-x}$

sign of f' : $\begin{array}{ccccccc} & - & - & 0 & + & + & + \\ \hline & -1 & 0 & 1 & 2 & & \end{array} \rightarrow x$

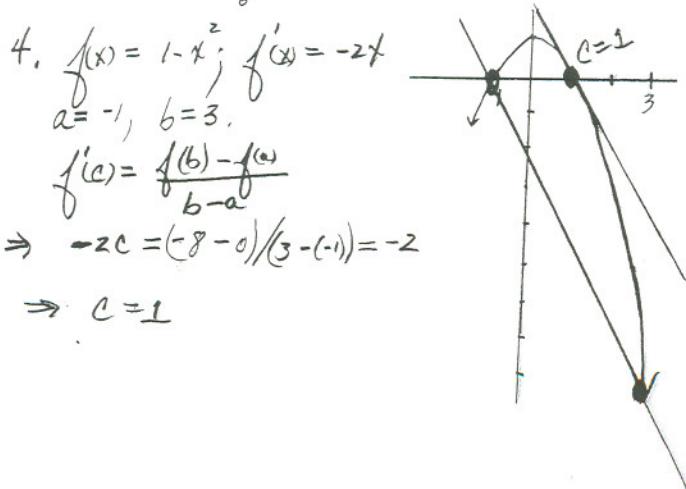
sign of f : $\begin{array}{ccccccc} + & + & + & 0 & - & - & - \\ \hline -1 & 0 & 1 & 2 & & & \end{array} \rightarrow x$

sign of f'' : $\begin{array}{ccccccc} - & - & - & 0 & + & + & + \\ \hline -3 & 0 & 1 & 2 & & & \end{array} \rightarrow x$



- (b) increases on $(-\infty, 1)$, decreases on $(1, \infty)$
 (c) concave up on $(2, \infty)$, down on $(-\infty, 2)$

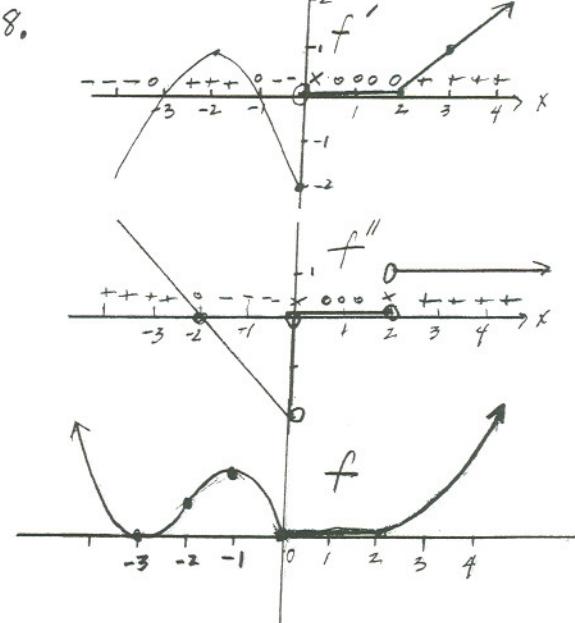
3. (a) $\lim_{x \rightarrow 1} \frac{x^2 + 2x + 1}{x+2} = \frac{4}{3}$; (b) $\lim_{x \rightarrow \pm} \frac{\ln(x)}{x} = 0$
 (c) $\lim_{x \rightarrow 1} \frac{\frac{d}{dx}(x)}{\ln(x)} \stackrel{H}{=} \lim_{x \rightarrow 1} \frac{1}{\frac{1}{x}} = 1$



5. (a) $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \frac{2}{-1} = -2$; (b) $\lim_{x \rightarrow 1} \frac{f'(x)}{g'(x)} = \lim_{x \rightarrow 1} \frac{f(x)}{g(x)} = \frac{0}{1} = 0$

6.
 $V = (9-2x)(12-2x) x; x \in [0, \frac{9}{2}]$
 $V(x) = 4x^3 - 42x^2 + 108x$
 $V'(x) = 12x^2 - 84x + 108 = 12(x^2 - 7x + 9) = 0$
 $\Rightarrow x = (7 - \sqrt{13})/2 \approx 1.70$ or $x = (7 + \sqrt{13})/2 \approx 5.30$
 $V''(x) = 24x - 84 \Rightarrow V''(1.70) < 0 \Rightarrow \text{max}$

7. $A(t) = 4t^2 \xrightarrow{\text{antideriv}} A(t) = \frac{4t^3}{3} + C$
 $A(0) = 5 \Rightarrow \frac{4(0)^3}{3} + C = 5 \Rightarrow C = 5 - \frac{4}{3} = \frac{11}{3}$
 $\Rightarrow A(t) = \frac{4}{3}t^3 + \frac{11}{3}$,



9. (a) $\int_0^2 f(x) dx = \int_0^1 x^2 dx + \int_1^2 f(x) dx$
 $= \frac{x^3}{3} \Big|_0^1 + 0 = \frac{1}{3}$

(b)
 $R_4 = [f(1.5) + f(2) + f(2.5) + f(3)] \cdot 0.5$
 $= [0.25 + 1 + 0 - 1] \cdot 0.5 = 0.125$

(c) $M_2 = [f(0.5) + f(1.5)][1] = 0.25$