Solutions to Chapter 3 Practice Problems   Calculus I

1. $y' = 8(x+2)^7(x+3)^6 + (x+2)^8\, 6(x+3)^5$
$= 2(x+2)^7(x+3)^5[4(x+3)+3(x+2)]$
$= 2(x+2)^7(x+3)^5[7x+18]$

2. $y' = (x^{1/3}+x^{-1/3})' = \frac{1}{3}x^{-2/3} - \frac{1}{3}x^{-4/3}$
$= \frac{1}{3}x^{-4/3}(x^{2/3}-1) = \frac{1}{3}\,\frac{\sqrt[3]{x^2}-1}{\sqrt[3]{x^4}}$

3. $y' = \frac{e^x(1+x^2)-e^x(2x)}{(1+x^2)^2} = \frac{e^x(x^2-2x+1)}{(1+x^2)^2}$

4. $y' = \cos(\cos(x))\cdot(-\sin(x))$

5. $y' = \frac{1}{\sqrt{1-(e^x)^2}}\cdot e^x = \frac{e^x}{\sqrt{1-e^{2x}}}$

6. $y' = 1e^{-1/x} + xe^{-1/x}\cdot(-1/x)'$
$= e^{-1/x} + xe^{-1/x}\left(\frac{1}{x^2}\right)$
$= e^{-1/x}\left(1+\frac{1}{x}\right)$

7. $y' = \sec^2((1-x)^{1/2})\cdot\frac{1}{2}(1-x)^{-1/2}(-1)$
$= \frac{-\sec^2\sqrt{1-x}}{2\sqrt{1-x}}$

8. $y = \frac{1}{\csc(5x)}\cdot(-\csc(5x)\operatorname{cotan}(5x))\cdot 5$
$= -5\operatorname{cotan}(5x)$

9. $y' = e^{e^x}\cdot e^x = e^{e^x+x}$

10. $y' = \frac{1}{1+(\arcsin\sqrt{x})^2}\cdot\frac{1}{\sqrt{1-(\sqrt{x})^2}}\cdot\frac{1}{2}x^{-1/2}$
$= \frac{1}{1+(\arcsin\sqrt{x})^2}\cdot\frac{1}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{x}}$

11. $y' = 5^{x\tan x}(\ln 5)\cdot(1\tan x + x\sec^2 x)$

12. $\ln y = x^2\ln x$
$\frac{1}{y}\frac{dy}{dx} = 2x\ln x + x^2\cdot\frac{1}{x}$
$\frac{dy}{dx} = x^{x^2}\cdot[2\ln x+1]x$

13. $\ln y = \frac{1}{2}\ln(x+1) + 5\ln(2-x) - 7\ln(x+3)$
$\frac{1}{y}y' = \frac{1/2}{x+1}\ \frac{+5}{2-x}(-1)\ \frac{-7}{x+3}$
$y' = \frac{\sqrt{x+1}\,(2-x)^5}{(x+3)^7}\left[\frac{1/2}{x+1} - \frac{5}{2-x} - \frac{7}{x+3}\right]$

14. $\frac{d}{dx}(x^{1/2}+y^{1/2}) = \frac{d}{dx}(3) \Rightarrow \frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' = 0$
$y' = \frac{-x^{-1/2}}{y^{-1/2}} = -\frac{\sqrt{y}}{\sqrt{x}}\Big|_{(4,1)} = -\frac{1}{2}$
tang line: $y-1 = -\frac{1}{2}(x-4)$.

15. $P'(x) = f'(x)g(x) + f(x)g'(x)$
$P'(2) = (5)(1) + (3)(4) = 5+12 = 17$
$Q'(x) = (f'(x)g(x) - f(x)g'(x))/g^2(x)$
$Q'(2) = ((5)(1) - (3)(4))/1^2 = -7$

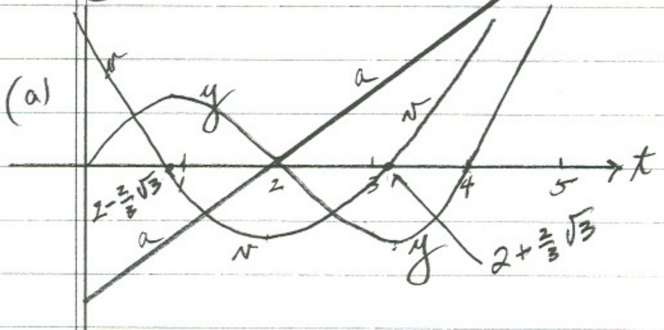
$C'(x) = f'(g(x))\cdot g'(x)$
$C'(2) = f'(g(2))\cdot g'(2)$
$= f'(1)\cdot 4$
$= (-2)(4) = -8$

16. $y(t) = t^3 - 6t^2 + 8t = t(t^2 - 6t + 8)$
$= t(t-2)(t-4)$

$v(t) = y'(t) = 3t^2 - 12t + 8$
roots $2 \pm \frac{2}{3}\sqrt{3}$

$a(t) = y''(t) = 6t - 12 = 6(t-2)$

(a)



(b) moving up $0 < t < 2 - \frac{2}{3}\sqrt{3}$
and $2 + \frac{2}{3}\sqrt{3} < t < 5$
moving down $2 - \frac{2}{3}\sqrt{3} < t < 2 + \frac{2}{3}\sqrt{3}$

(c) dist: $\left|y(2-\frac{2}{3}\sqrt{3}) - y(0)\right|$
$+ \left|y(2+\frac{2}{3}\sqrt{3}) - y(2-\frac{2}{3}\sqrt{3})\right|$
$+ \left|y(5) - y(2+\frac{2}{3}\sqrt{3})\right|$
$= \frac{16\sqrt{3}}{9} + \frac{32\sqrt{3}}{9} + \frac{15 + 16\sqrt{3}}{9}$
$= \frac{64\sqrt{3}}{9} + 15 \doteq 27.3$

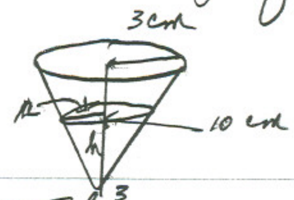
17. $V = \pi r^2 h$

(a) $\frac{dV}{dh} = \pi r^2 \cdot 1 \quad \left(\frac{m^3}{m} = m^2\right)$

(b) $\frac{dV}{dr} = 2\pi r h \quad \left(\frac{m^3}{m} = m^2\right)$

(c) $V(t) = \pi (r(t))^2 h(t)$
$\frac{dV}{dt} = \pi\left(2(r(t))\frac{dr}{dt} h(t) + (r(t))^2 \frac{dh}{dt}\right)$
$= \pi(2rhr' + r^2h') \quad \left(m^3/s\right)$

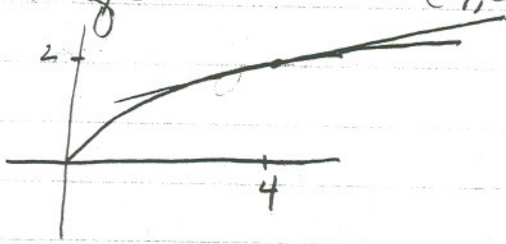
18. $V = \frac{1}{3}\pi r^2 h$



$\frac{r}{h} = \frac{3}{10}$
$\Rightarrow r = .3h$
$V = \frac{1}{3}\pi(.3h)^2 h = .03\pi h^3$
$\frac{dV}{dt} = .09\pi h^2 \cdot dh/dt$

$2\,cm^3/s = .09\pi(5\,cm)^2 \cdot \frac{dh}{dt}$
$\Rightarrow \frac{dh}{dt} = \frac{2\,cm^3/s}{.09\pi\, 25\,cm^2} = \frac{8}{9\pi}\,cm/s$

19. $f(x) = x^{1/2} \quad a = 4 \quad (4, 2)$



$y - 2 = m(x-4) \quad m = f'(4)$
$m = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}\Big|_4 = \frac{1}{4}$
linearization: $y = \frac{1}{4}(x-4) + 2$
$y = \frac{1}{4}x + 1$
$\sqrt{4.03} \doteq \frac{1}{4}(4.03) + 1 = 2.0075$
too high because curve is concave down.

20. $\cancel{y = Ae^{-x} + Bxe^{-x}}$
$2[\cancel{y' = (-A+B)e^{-x} - Bxe^{-x}}]\,2$
$\cancel{y'' = (A-2B)e^{-x} + Bxe^{-x}}$

$y'' + 2y' + y = 0$ ✓