

6 Week Exam  
SM121, Section 2043  
Calculus 1

9 September 2008

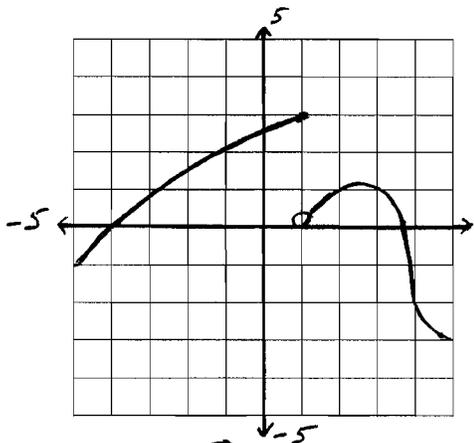
Instructions

1. Do not open the exam until instructed.
2. The exam will commence immediately at the beginning of class, or when the entire class is present, whichever occurs first.
3. You may not use any notes, books, calculators, or other materials.
4. You must use pencil only.
5. You may not discuss the exam with anyone, using any medium, until I explicitly give you permission to do so!
6. Full credit will only be granted for showing a logical progression of all steps leading to the correct answer.
7. Graphs must be properly labeled.
8. Simple arithmetic must be computed for full credit. Answers should be clear and complete without further substitutions or re-arrangement.
9. Draw a box around your final answer.
10. Turn in all scratch paper with your name on each sheet.

Name: \_\_\_\_\_

*Solution*

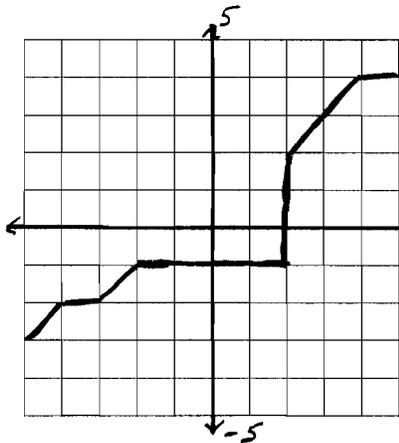
1. (9 pts) Is the given curve a graph of a function of  $f$ ? If it is, state the domain and range.



Function:  Y /  N

Domain:  $[-5, 5]$

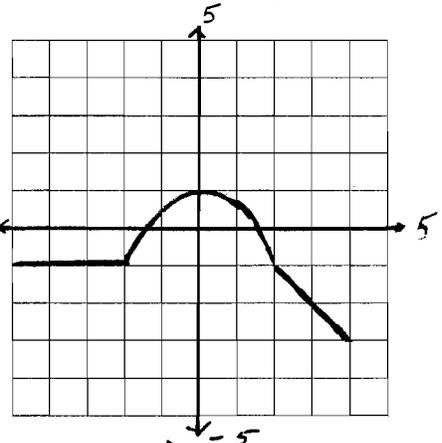
Range:  $[-3, 3]$



Function: Y /  N

Domain: \_\_\_\_\_

Range: \_\_\_\_\_

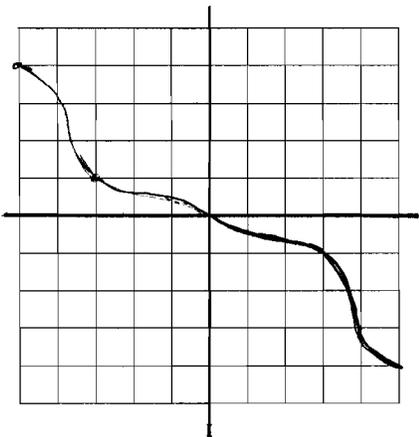


Function:  Y /  N

Domain:  $[-5, 4]$

Range:  $[-3, 1]$

2. (4 pts) The function  $f$  has domain  $[-4, 4]$  and a portion of the graph is shown. Complete the graph of  $f$  if it is known that  $f$  is odd.



$f$  is odd, therefore

$$f(-x) = -f(x)$$

symmetric about the origin

$x$	3	4	5
$f(x)$	-1	-3.2	-4
$-f(x)$	1	3.2	4
$f(-x)$	1	3.2	4

equal ↗

3. (6 pts) Find the equation of the line passing through the point  $(2, 3)$  that is parallel to the line  $y=3x+5$ .

$$m = 3$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 3(x - 2)$$

$$y - 3 = 3x - 6$$

$$\boxed{y = 3x - 3} \quad \text{or} \quad \boxed{y - 3 = 3(x - 2)}$$

4. (10 pts) An empty dump truck weighs 6,000 pounds. Sand is added to the truck at a rate of 170 pounds per minute.

a) Express the total weight of the truck (truck plus sand) as a function of time.

$$f(t) = 170t + 6000$$

b) How long will it take (in minutes) for the truck to weigh 8,750 pounds?

$$8750 = 170t + 6000$$

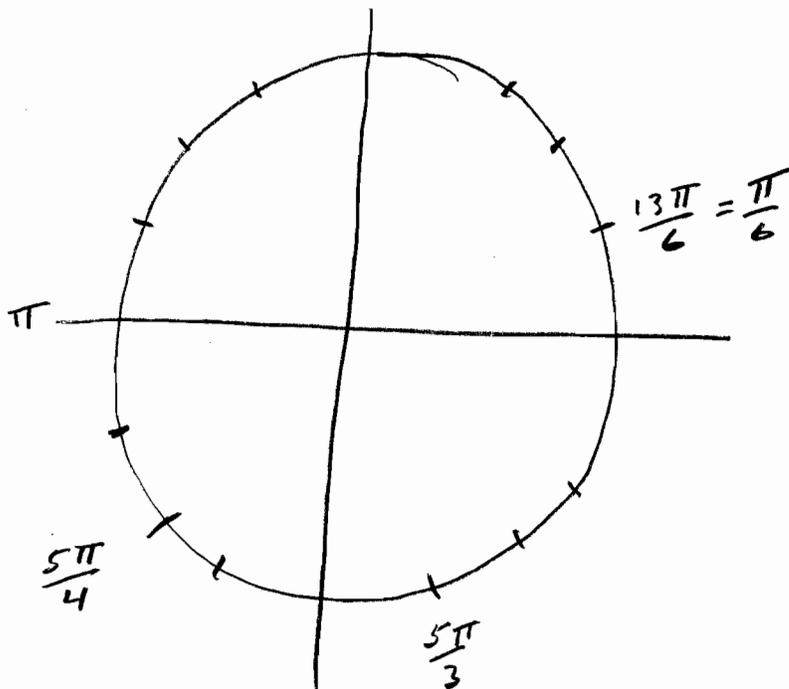
$$2750 = 170t$$

$$\frac{2750}{170} = t$$

$$t \approx 16.18 \text{ min}$$

5. (8 pts) Fill in the following table:

$\theta$	$\pi$	$\frac{5\pi}{4}$	$\frac{13\pi}{6}$	$\frac{5\pi}{3}$
$\sin \theta$	0	$-\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$\cos \theta$	-1	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$



6. (10 pts) Find  $f \circ g \circ h\left(\frac{1}{3}\right)$  given the following function definitions;

$$f(x) = \cos\left(\frac{\pi}{3}x\right) \quad g(x) = x^2 \quad h(x) = 3x + 1$$

$$(f \circ g \circ h)\left(\frac{1}{3}\right) = f(g(h(x))) \text{ at } x = \frac{1}{3}$$

$$g(h(x)) = (3x+1)^2$$

$$= 9x^2 + 6x + 1$$

$$f(g(h(x))) = \cos\left(\frac{\pi}{3}(9x^2 + 6x + 1)\right) \text{ at } x = \frac{1}{3}$$

$$= \cos\left(\frac{\pi}{3}\left(9 \cdot \frac{1}{9} + \frac{6}{3} + 1\right)\right)$$

$$= \cos\left(\frac{\pi}{3}(1 + 2 + 1)\right)$$

$$= \cos\left(\frac{4\pi}{3}\right)$$

$$(f \circ g \circ h)\left(\frac{1}{3}\right) = -\frac{1}{2}$$

7. (9 pts) Write the equations of the graphs that are obtained from the graph of  $y = \sin x$

- a) shifted 4 units right

$$y = \sin(x - 4)$$

- b) shifted down 2 units

$$y = \sin x - 2$$

- c) stretched vertically by a factor of 3

$$y = 3 \sin x$$

8. (5 pts) What is the domain of the function  $f(x) = \frac{1}{3 - e^{2x}}$ ?

Domain is all real numbers except where denominator equals zero.

$$3 - e^{2x} = 0$$

$$e^{2x} = 3$$

$$\ln e^{2x} = \ln 3$$

$$2x = \ln 3$$

$$x = \frac{\ln 3}{2}$$

$$\therefore \text{Domain: } \left\{ x \in \mathbb{R} \mid x \neq \frac{\ln 3}{2} \right\}$$

9. (5 pts) Find the inverse function of  $f(x) = \frac{4x-1}{2x+3}$

$$y = \frac{4x-1}{2x+3}$$

$$(2x+3)y = 4x-1$$

$$2xy + 3y = 4x-1$$

$$2xy - 4x = -3y-1$$

$$x(2y-4) = -3y-1$$

$$x = \frac{-3y-1}{2y-4}$$

$f^{-1}(x) = \frac{-3x-1}{2x-4}$

10. (5 pts) Find the inverse function of  $f(x) = 2 \ln(3+x)$

$$y = 2 \ln(3+x)$$

$$\frac{y}{2} = \ln(3+x)$$

$$e^{y/2} = e^{\ln(3+x)}$$

$$e^{y/2} = 3+x$$

$$x = e^{y/2} - 3$$

$f^{-1}(x) = e^{x/2} - 3$

11. (7 pts) Simplify the following

a)  $a^4 a^{-2} = a^{4-2}$   
 $= \boxed{a^2}$

b)  $\frac{a^3 b^2 c^3}{a^{-1} b^3 c^{-3}} = a^3 a^1 b^2 b^{-3} c^3 c^3$   
 $= a^4 b^{-1} c^6$   
 $= \boxed{\frac{a^4 c^6}{b}}$

12. (5 pts) Solve the following equation for x;  $2^{x-5} = 3$

$$2^{x-5} = 3$$

$$\log_2 2^{x-5} = \log_2 3$$

$$x-5 = \log_2 3$$

$x = \log_2 3 + 5$

13. (12 pts) Find the exact value of the following expressions

$$\text{a) } \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

$$\text{b) } \tan^{-1}(1) = \pi/4$$

$$\begin{aligned} \text{c) } \log_4 128 + \log_4 5 - \log_4 10 &= \log_4 128 \cdot 5 - \log_4 10 \\ &= \log_4 \frac{128 \cdot 5}{10} &= \log_4 4^3 \\ &= \log_4 64 &= \boxed{3} \end{aligned}$$

14. (10 pts) Find the exponential function of the form  $y = Ca^x$  passing through the points (2, 3) and (1, 9).

$$f(x) = Ca^x \quad \begin{array}{l} f(1) = 9 \\ f(2) = 3 \end{array}$$

$$9 = Ca^1$$

$$C = 9/a$$

$$3 = \frac{9}{a} \cdot a^2$$

$$3 = \frac{9a^2}{a}$$

$$3 = 9a$$

$$\boxed{a = 1/3}$$

$$\rightarrow 9 = C\left(\frac{1}{3}\right)^1$$

$$9 \cdot 3 = C$$

$$\boxed{C = 27}$$

$$\therefore \boxed{f(x) = 27 \cdot \left(\frac{1}{3}\right)^x}$$