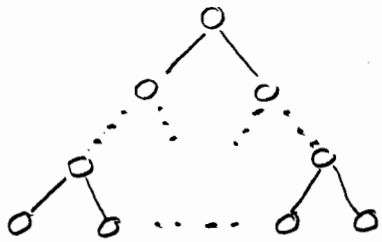
# nodes		# levels from leaf to root
1		$\lfloor \log n \rfloor$
		$\vdots$
$n/4$		1
$n/2$		0



Total time = time to collect n elements + time to heapify

$O(n)$ to collect n elements

Time to heapify
- 0 swaps to build $n/2$ heaps
- 1 swap to build $n/4$ heaps
- 2 swaps to build $n/8$ heaps
- $\vdots$
- $\lfloor \log n \rfloor$ swaps to build/fix final heap.

$$\text{Time to heapify} = 0 \cdot n/2 + 1 \cdot n/4 + \cdots + \lfloor \log n \rfloor \cdot 1$$

$$= \sum_{i=0}^{\lfloor \log n \rfloor} i \frac{n}{2^{i+1}}$$

$$= n \sum_{i=0}^{\lfloor \log n \rfloor} i \left(\frac{1}{2^{i+1}}\right)$$

$$= n \sum_{i=0}^{\lfloor \log n \rfloor} i \left(\frac{1}{2}\right)^{i+1}$$

$$= n \cdot \frac{1}{2} \underbrace{\sum_{i=0}^{\lfloor \log n \rfloor} i \left(\frac{1}{2}\right)^{i}}$$

Aside:

$$\sum_{k=0}^{\infty} k x^k = \frac{x}{(1-x)^2} \quad \text{for } |x|<1$$

$$\sum_{i=0}^{\lfloor \log n \rfloor} i \left(\frac{1}{2}\right)^{i} \le \sum_{i=0}^{\infty} i \left(\frac{1}{2}\right)^{i}$$

$$= \frac{\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2}$$

$$= 2$$

$$\sum_{i=0}^{\lfloor \log n \rfloor} i \left(\frac{1}{2}\right)^{i} \le 2$$

$$\le n \cdot \frac{1}{2} \cdot 2$$

$$\le n$$

$\therefore$ Time to heapify $\le n = O(n)$

$$\boxed{\text{Total time} = O(n) + O(n) = O(n)}$$

3-0235 — 50 SHEETS — 5 SQUARES
3-0236 — 100 SHEETS — 5 SQUARES
3-0237 — 200 SHEETS — 5 SQUARES
3-0137 — 200 SHEETS — FILLER
COMET