

6 Week Exam  
IC-312, Data Structures  
Fall AY08

September 24, 2007

**This is a multi section exam that will be given to different midshipmen at different times. As per USNAINST 1531.53A, you may NOT communicate about this exam with anyone using any medium until your instructor tells you that you can.**

Unless otherwise specified, all Java source code given in this exam is syntactically correct.

Name: Solution

~~59~~  
59

- 3 1. What is the result of executing the following method if the method is invoked with `alg1(6)`?

```
public int alg1(int n) {  
    int sum = alg1(n-3) + n + alg1(n-2);  
    if (n <= 0) return 0;  
    return sum;  
}
```

17

- 3 2. Will the following recursive method eventually return a result, or will it enter an infinite recursive loop that terminates in a stack overflow? Justify your answer.

```
public static int mystery(int a, int b) {  
    if (b == 0)  
        return 0;  
    if (b%2 == 0)  
        return mystery (2*a, b/2);  
    return mystery(2*a, b/2) + a;  
}
```

Yes

The base case occurs when  $b=0$ . Both calls to `mystery` cut  $b$  in half. Because of integer division  $b$  will eventually equal zero.

- 4 3. True or False
- $3n^2 + \log n$  is  $O(n^3)$  ..... True or False
  - $5n \log n + 3n$  is  $O(n \log n)$  ..... True or False
  - $5n \log n + 3n$  is  $\Theta(n \log n)$  ..... True or False
  - $5n \log n + 3n$  is  $\Omega(n \log n)$  ..... True or False

- 2 4. Use the definition of *big-O* to show that  $6n^2+3$  is  $O(n^2)$ .  
Note: You need to find  $c > 0$  and  $n_0 \geq 1$ .

$$6n^2 + 3 \leq cn^2 \quad \forall n \geq n_0$$

$$6n^2 + 3 \leq 6n^2 + 3n^2 \quad \forall n \geq 1$$

$$\leq 9n^2$$

$$\therefore c = 9 \quad n_0 = 1$$

- 2 5. Use the definition of  $\Omega$  to show that  $n^2 \log n$  is  $\Omega(n^2)$ .  
Note: You need to find  $c > 0$  and  $n_0 \geq 1$ .

$$n^2 \log n \geq cn^2 \quad \forall n \geq n_0 \quad c > 0$$

$$\frac{n^2 \log n}{n^2} \geq \frac{cn^2}{n^2}$$

$$\log n \geq c$$

if  $n \geq 2$ , then  $\log n$  will be  $\geq 1$   
 $\therefore$  let  $n_0 = 2$  &  $c = 1$

- 3 ~~4~~ 6. Using big-O notation, indicate the time requirement of each of the following tasks in the worst case.
- You search for a parking place in a parking lot that has  $n$  rows of parking, with  $m$  parking places in each row.  $O(m \cdot n)$
  - You search for the perfect pineapple to purchase, but instead of looking at every pineapple in the produce market, you inspecting every third pineapple. There are a total of  $n$  pineapples at the market.  $O(n)$
  - Visit every node in a general tree of height  $h$ , where each node level of the tree has  $k$  nodes.  $O(k \cdot h)$

- (2) d. Extra credit: You search for a name in the phone book using a binary search technique. ( $n$  is the total number of names in the phone book).

$$O(\log n)$$

2 7. List the following five functions in order from smallest to largest:

$n^2/\log n$ ,  $\ln e^{(n^2)}$ ,  $\sqrt{n}$ ,  $3^n$ ,  $n^2 + \log n$ ,  $2^{15}$   
3      4      2      6      5      1

$$2^{15} \leq \sqrt{n} \leq \frac{n^2}{\log 4} \leq \ln e^{(n^2)} \leq n^2 + \log n \leq 3^n$$

2 8. What are the running times of the following methods in terms of *big-O*?

a. 

```
public static int alg2(int k) {
    int sum = 0;
    k = k*2;
    for (int i = 0; i < k; i++)
        for (int j = 0; j < i; j++)
            sum += (i + j);
    return sum;
}
```

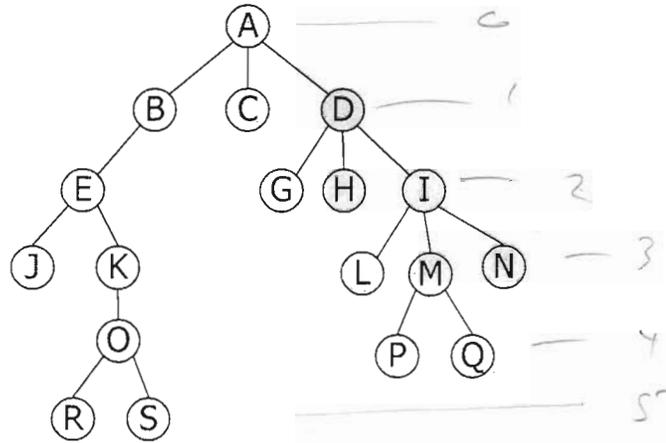
$$O(k^2)$$

b. 

```
public static int alg3(int n) {
    int product = 1;
    for (int i = 1; i < n/2; i++)
        product *= i;
    return product;
}
```

$$O(n)$$

9. The following questions refer to the tree shown below:



- a. How many internal nodes are there? **8**
- b. List the external nodes. **J R S C G H L P Q N**
- c. What is the height of the tree? **5**
- d. What is the depth of node L? **3**
- e. List the ancestors of node K? **E B A**
- f. List the descendants of node I. **L M N P Q**
- g. List the siblings of node D. ~~A B C~~ **B C**
- h. What are the nodes in the sub-tree rooted at K? **O R S** or **K O R S**

10. Suppose a general tree represents the organizational hierarchy of a company. Every level represents a "rank" in the organization with the root being the topmost rank. You may assume the tree has height  $h$ . Each node has at least  $r$  children and at most  $k$  children.

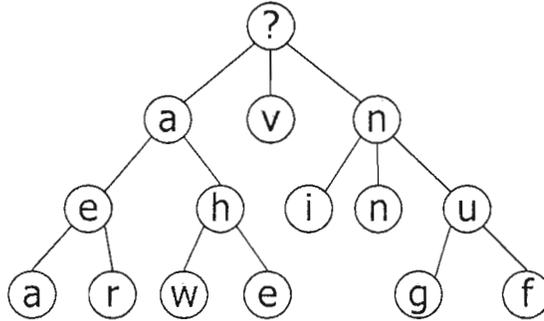
- a. What is the minimum number of nodes in the organization tree?

$$\sum_{i=0}^h r^i = \frac{r^{h+1} - 1}{r - 1}$$

- b. Describe in words an algorithm for determining the actual number of ranks in the organization.

Run the ~~depth~~ **height** algorithm using the root as the starting node. The number of ranks is height + 1.

11. Given the following tree, list the order in which the nodes would be visited during a pre-order traversal and post-order traversal.



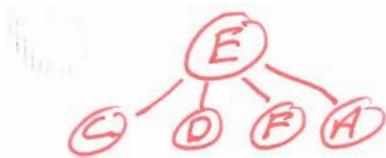
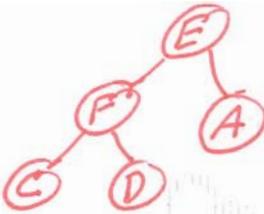
2 a. Pre-order traversal  
? a e a r h w e v n i n u g f

2 b. Post-order traversal  
a r e w e h a r i n g u n v ?

12. Assume you have a general tree with 5 nodes labeled A thru E and a post-order traversal of the tree results in the nodes being visited in the following order (C, D, F, A, E).

1 True or False: The post order traversal uniquely defines the shape of the tree, i.e., there is only one tree that will yield the above visit order when traversed in a post-order fashion.

2 Justify your answer.



Both of the above trees, when traversed with a post order traversal, will result in the nodes being visited in the same order. (C D F A E)

13. We discussed two implementations of the List ADT; a linked-list and an array. Each has its own advantages and drawbacks. Describe the advantages and disadvantages of both the linked-list and array-based implementation of the List ADT.

3 Linked-list

Advantage:

No limit on number of elements (unless physical memory constraints)

Disadvantage:

$O(n)$  to find elements unless they are at the beginning or end of the list. Even if you know the element's index is the list.

3 Array

Advantage:

$O(1)$  access to elements in the middle of the list.

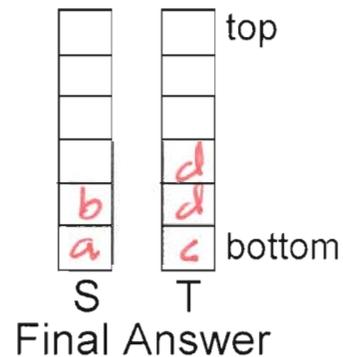
Disadvantage:

Finite storage space.

3 14. Suppose  $s$  and  $t$  are empty stacks and  $a, b, c, d$  and  $e$  are objects. What do the stacks contain after the following sequence of operations?

```
s.push(a)
s.push(b)
t.push(c)
t.push(d)
t.push(e)
s.push(t.pop())
s.push(t.peek())
t.push(s.pop())
s.pop()
```

<del>d</del>	d
<del>e</del>	e
b	d
a	c
s	t

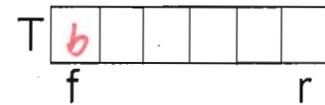
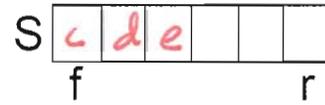


- 3 15. Suppose  $s$  and  $t$  are empty queues and  $a, b, c, d$  and  $e$  are objects. What do the queues contain after the following sequence of operations?

```

s.enqueue(a)
s.enqueue(b)
s.enqueue(c)
t.enqueue(d)
t.enqueue(e)
s.enqueue(t.dequeue())
s.dequeue()
t.enqueue(s.front())
s.enqueue(t.dequeue())
s.dequeue()

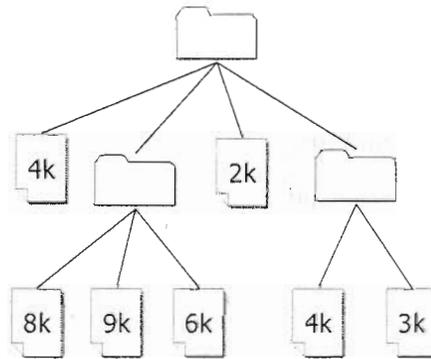
```



Final Answer

s: ~~a~~ b c d e  
t: ~~d~~ e b

16. In most operating systems, files are organized hierarchically in nested directories or folders which are presented in the form of a tree. The internal nodes of the tree are directories and the external nodes are files. The files have an associated size which indicates the amount of space they occupy.



- 3 a. Describe in pseudocode an efficient algorithm for summing up the total space used by all of the files in a directory, including the files in any nested subdirectories. For example, running your algorithm on the directory above should return a value of 36. Do not make assumptions about the size or shape of the tree; your algorithms should work on any general tree.

```

dirSize(Node v)
  let totalSize = 0
  if v is external
    return v.size
  else
    for each child w of v
      totalSize += dirSize(w)
  return totalSize

```

Use a post order traversal to visit the nodes of the subtree on tree. If the node is external it is a file and will have a size. Accumulate the size of all the leaf nodes.

- 1 b. What is the worst case run time of your algorithm as a function of  $n$ , where  $n$  is the number of nodes in the tree (files & directories). Express your answer in terms of big-O.

$O(n)$  - the algorithm must traverse each of the  $n$  nodes to total up the total size of