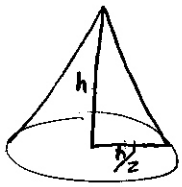
SM 121 LESSON 35 27/30/31/34 10/22/2008

SECTION 3.9 RELATED RATES CONTINUED

(27)



$V = \frac{1}{3}\pi r^2 h$

$r = \frac{h}{2}$

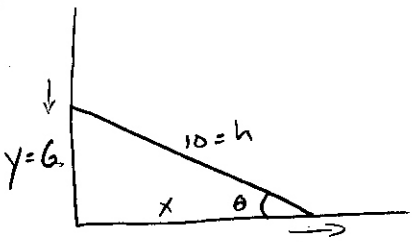
$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$ $\quad h = 10$ ft $\quad \frac{dV}{dt} = 30$ ft^3/min

$\frac{d}{dt}\left(V = \frac{\pi h^3}{12}\right)$ $\quad \frac{dh}{dt} = ?$

$\frac{dV}{dt} = \frac{\pi}{12} 3h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{\frac{\pi}{4}h^2} = \frac{30}{\frac{\pi}{4}(10)^2} = \frac{120}{100\pi} = \frac{6}{5\pi}$ ft/min $\boxed{\approx 0.382 \text{ ft/min}}$

(30)



$\frac{dx}{dt} = 1$ ft/s $\quad \frac{dy}{dt} = ?$

$x = 8$ ft

$y^2 + x^2 = 100$

$x^2 = 100 - y^2$

$x = \sqrt{100 - y^2} = \sqrt{100 - 6^2} = \sqrt{100 - 36} = \sqrt{64} = 8$

$\frac{d}{dt}\left(y^2 + x^2 = 100\right)$

$2y\frac{dy}{dt} + 2x\frac{dx}{dt} = 0$

$\cancel{2}y\frac{dy}{dt} = -\cancel{2}x\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{-x\frac{dx}{dt}}{y} = -\frac{8}{6}(1) = -\frac{4}{3}$ ft/s

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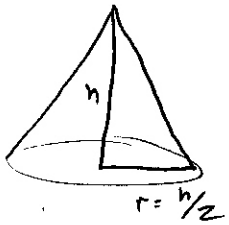
SECTION 3.9 RELATED RATES CONTINUED

(27)



$V = \frac{1}{3}\pi r^2 h$ $\quad h = 10$ ft $\quad \frac{dV}{dt} = 30$ ft^3/min

$r = \frac{h}{2}$ $\quad \frac{dh}{dt} = ?$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

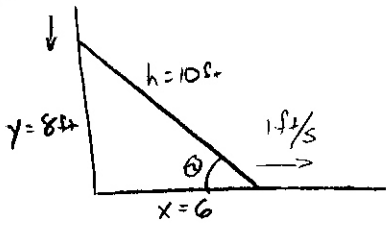
$V = \frac{1}{3}\pi \frac{h^2}{4} h$

$\frac{d}{dt}\left(V = \frac{\pi h^3}{12}\right)$

$\frac{dV}{dt} = \frac{\pi}{4\cancel{12}}\left(\cancel{3}h^2 \frac{dh}{dt}\right) = \frac{\pi h^2}{4}\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{4}{\pi h^2}\frac{dV}{dt} = \frac{4}{\pi(10)^2}(30) = \frac{120}{100\pi} = \frac{6}{5\pi}$ ft/min $\boxed{\approx 0.382 \text{ ft/min}}$

(30)



$x = 6$ ft $\quad \frac{dx}{dt} = 1$ ft/s

$y^2 + 6^2 = 10^2$

$y^2 + 36 = 100$

$y^2 = 64$

$y = \sqrt{64} = 8$ ft

$\cos\theta = \frac{x}{h}$

$\frac{d}{dt}\left(\cos\theta = \frac{x}{10}\right)$

$-\sin\theta \frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt}(-\csc\theta)$ $\quad \csc\theta = \frac{h}{y} = \frac{10}{8} = \frac{5}{4}$

$\frac{d\theta}{dt} = \frac{1}{\cancel{10}\,2}(1)\left(-\frac{8}{4}\right) = \boxed{-\frac{1}{8} \text{ rad/s}}$

LESSON 35 10/22/2008

(21) $\frac{d}{dt}(PV = C)$

$P = 150\,kPa$ $\frac{dP}{dt} = 20\,kPa/min$

$V = 600\ cm^3$ $\frac{dV}{dt} = ?$

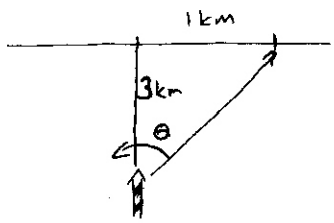
$$P\frac{dV}{dt} + V\frac{dP}{dt} = 0$$

$$P\frac{dV}{dt} = -V\frac{dP}{dt}$$

$$\frac{dV}{dt} = \frac{-V}{P}\frac{dP}{dt} = \frac{-\cancel{600}^{4}}{\cancel{150}}(20) = -4(20) = -80\ cm^3/min$$

$$\boxed{\text{VOLUME IS DECREASING AT } 80\ cm^3/min}$$

(38)



$x = 1\,km$ $\frac{dx}{dt} = ?$

$y = 3\,km$

$h = \sqrt{10}\,km$ $\frac{d\theta}{dt} = 8\pi\ rad/min$

$$x^2 + y^2 = h^2$$

$$1^2 + 3^2 = h^2$$

$$1 + 9 = h^2$$

$$h = \sqrt{10}$$

$$\tan\theta = \frac{x}{y}$$

$$\frac{d}{dt}\left(\tan\theta = \frac{x}{3}\right)$$

$$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{3}\frac{dx}{dt}$$

$$\frac{dx}{dt} = 3\sec^2\theta\frac{d\theta}{dt} \qquad \sec\theta = \frac{h}{y} = \frac{\sqrt{10}}{3}$$

$$\frac{dx}{dt} = 3\left(\frac{\sqrt{10}}{3}\right)^2(8\pi)$$

$$\frac{dx}{dt} = \cancel{3}\left(\frac{10}{\cancel{9}_3}\right)(8\pi) = \frac{80\pi}{3}\ km/min \quad \boxed{\approx 83.78\ km/min}$$