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SECTION 3.5 IMPLICIT DIFFERENTIATION

② $\frac{d}{dx}(4x^2 + 9y^2 = 36)$

$$\frac{d}{dx}(4x^2) + \frac{d}{dx}(9y^2) = \frac{d}{dx}(36)$$

$$8x + 18y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-8x}{18y}$$

ⓐ $\boxed{\frac{dy}{dx} = \frac{-4x}{9y}}$

$$4x^2 + 9y^2 = 36$$

$$9y^2 = 36 - 4x^2$$

$$y^2 = \frac{36 - 4x^2}{9}$$

$$y = \sqrt{\frac{4}{9}(9 - x^2)}$$

$$y = \pm\frac{2}{3}\sqrt{9 - x^2}$$

$$y = \pm\frac{2}{3}(9 - x^2)^{\frac{1}{2}}$$

$$y' = \pm\frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{2}}(9 - x^2)^{-\frac{1}{2}}(-2x)$$

ⓑ $\boxed{y' = \mp\frac{2x}{3(9-x)^{\frac{1}{2}}}}$

$$\frac{dy}{dx} = \frac{-4x}{\cancel{9}^{3}\left(\pm\frac{2}{\cancel{3}}(9 - x^2)^{\frac{1}{2}}\right)}$$

$$= \frac{-4x}{\pm 6\sqrt{9 - x^2}}$$

$\boxed{\frac{dy}{dx} = \mp\frac{2x}{3\sqrt{9 - x^2}}}$ MATCHES PART B

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(7) $\frac{d}{dx}(x^2 + xy - y^2 = 4)$

$$\frac{d}{dx}x^2 + \frac{d}{dx}xy - \frac{d}{dx}y^2 = \frac{d}{dx}4$$

$$2x + (1)y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$$

$$xy' - 2yy' = -2x - y$$

$$y'(x - 2y) = -2x - y$$

$$y' = \frac{-(2x+y)}{(x-2y)}$$

$$\boxed{y' = \frac{2x+y}{2y-x}}$$

(16) $\frac{d}{dx}(y^5 + x^2y^3 = 1 + ye^{x^2})$

$$\frac{d}{dx}y^5 + \frac{d}{dx}x^2y^3 = \frac{d}{dx}1 + \frac{d}{dx}ye^{x^2}$$

$$5yy' + 2xy^3 + 2x^2y^2y' = 0 + y'e^{x^2} + ye^{x^2}(2x)$$

$$5yy' + 2x^2y^2y' - y'e^{x^2} = 2xye^{x^2} - 2xy^3$$

$$y'(5y + 2x^2y^2 - e^{x^2}) = 2xy(e^{x^2} - y^2)$$

$$\boxed{y' = \frac{2xy(e^{x^2} - y^2)}{5y + 2x^2y^2 - e^{x^2}}}$$

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(12) $\frac{d}{dx}\left(1 + x = \sin(xy^2)\right)$

$\frac{d}{dx}1 + \frac{d}{dx}x = \frac{d}{dx}\sin(xy^2)$

$0 + 1 = \cos(xy^2)\left(\frac{d}{dx}xy^2\right)$

$1 = \cos(xy^2)\left((1)y^2 + (x)(2yy')\right)$

$1 = \cos(xy^2)(y^2 + 2xyy')$ OR $1 = \cos(xy^2)(y^2 + 2xyy')$

$\frac{1}{\cos(xy^2)} = y^2 + 2xyy'$

$\sec(xy^2) - y^2 = 2xyy'$

$$\boxed{y' = \frac{\sec(xy^2) - y^2}{2xy}}$$

$1 = y^2\cos(xy^2) + 2xy\cos(xy^2)y'$

$2xy\cos(xy^2)y' = 1 - y^2\cos(xy^2)$

$$\boxed{y' = \frac{1 - y^2\cos(xy^2)}{2xy\cos(xy^2)}}$$

(21) IF $f(x) + x^2[f(x)]^3 = 10$ $f(1) = 2$ FIND $f'(1)$

$\frac{d}{dx}\left(f(x) + x^2[f(x)]^3 = 10\right)$

$f'(x) + 3x^2[f(x)]^2 f'(x) + 2x[f(x)]^3 = 0$

$f'(1) + 3(1)^2(f(1))^2(f'(1)) + 2(1)(f(1))^3$

$f'(1) + 3(2)^2 f'(1) + 2(2)^3 = 0$

$f'(1) + 12f'(1) + 16 = 0$

$13f'(1) = -16$

$$\boxed{f'(1) = \frac{-16}{13}}$$

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(29) $2(x^2+y^2)^2 = 25(x^2-y^2)$ TANGENT AT $(3,1)$

$$\frac{d}{dx}\left(2(x^2+y^2)^2 = 25(x^2-y^2)\right)$$

$$2\frac{d}{dx}(x^2+y^2)^2 = 25\frac{d}{dx}(x^2-y^2)$$

$$2\left(2(x^2+y^2)\frac{d}{dx}(x^2+y^2)\right) = 25\left(\frac{d}{dx}x^2 - \frac{d}{dx}y^2\right)$$

$$4(x^2+y^2)\left(\frac{d}{dx}x^2 + \frac{d}{dx}y^2\right) = 25(2x - 2yy')$$

$$4(x^2+y^2)(2x + 2yy') = 25(2x - 2yy')$$

PLUG IN $(3,1)$

$$4(3^2+1^2)(2(3) + 2(1)y') = 25(2(3) - 2(1)y')$$

$$4(9+1)(6+2y') = 25(6-2y')$$

$$40(6+2y') = 150 - 50y'$$

$$240 + 80y' = 150 - 50y'$$

$$130y' = -90$$

$$y' = \frac{-90}{130}$$

$$y' = \frac{-9}{13}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{-9}{13}(x-3)$$

$$y - 1 = \frac{-9x}{13} + \frac{27}{13}$$

$$y = \frac{-9x}{13} + \frac{27}{13} + \frac{13}{13}$$

$$\boxed{y = \frac{-9x}{13} + \frac{40}{13}}$$