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SECTION 3.3 TRIGONOMETRIC DERIVATIVES CONTINUED

(21) TANGENT AT $\left(\frac{\pi}{3}, 2\right)$

$y = \sec x$

$y' = \sec x \tan x$

$y'\left(\frac{\pi}{3}\right) = \left(\sec\frac{\pi}{3}\right)\left(\tan\frac{\pi}{3}\right)$

$= (2)(\sqrt{3})$

$y'\left(\frac{\pi}{3}\right) = 2\sqrt{3}$

$y - y_1 = m(x - x_1)$

$y - 2 = 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$

$y - 2 = 2\sqrt{3}x - \frac{2\pi\sqrt{3}}{3}$

$\boxed{y = 2\sqrt{3}x + 2 - \frac{2\pi}{3}\sqrt{3}}$

(25) (a) TANGENT AT $\left(\frac{\pi}{2}, \pi\right)$

$y = 2x \sin x$

$y' = 2\sin x + 2x\cos x$

$y'\left(\frac{\pi}{2}\right) = 2\sin\frac{\pi}{2} + 2\left(\frac{\pi}{2}\right)\cos\frac{\pi}{2}$

$= 2(1) + \pi(0)$

$y'\left(\frac{\pi}{2}\right) = 2$

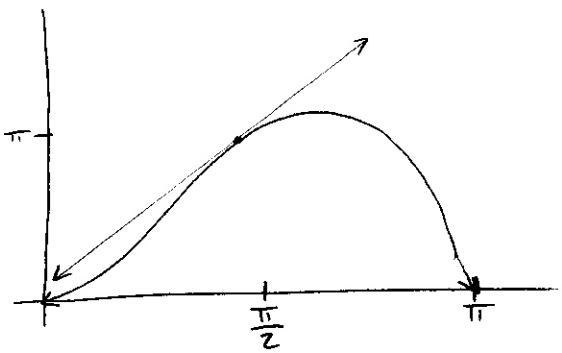
$y - y_1 = m(x - x_1)$

$y - \pi = 2\left(x - \frac{\pi}{2}\right)$

$y - \pi = 2x - \pi$

$\boxed{y = 2x}$

(b)



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(33) FOR WHAT VALUES OF x DOES $f(x) = x + 2\sin x$ HAVE A HORIZONTAL TANGENT?

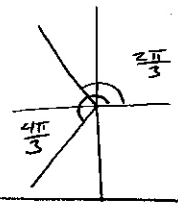
$$f'(x) = 1 + 2\cos x$$

$$1 + 2\cos x = 0$$

$$2\cos x = -1$$

$$\cos x = -\frac{1}{2}$$

$\cos x = -\frac{1}{2}$



PLUS n COMPLETE REVOLUTIONS

$\therefore$ $f(x)$ HAS HORIZONTAL TANGENTS WHENEVER $x = \frac{2\pi}{3} + 2\pi n$ AND $x = \frac{4\pi}{3} + 2\pi n$, WHERE n IS AN INTEGER

(35) $x(t) = 8\sin t$

(a)

$$v(t) = 8\cos t$$

$$a(t) = -8\sin t$$

(b)

$$x\left(\frac{2\pi}{3}\right) = 8\sin\frac{2\pi}{3} = 8\left(\frac{\sqrt{3}}{2}\right) = 4\sqrt{3} \text{ cm}$$

$$v\left(\frac{2\pi}{3}\right) = 8\cos\frac{2\pi}{3} = 8\left(-\frac{1}{2}\right) = -4 \text{ cm/s}$$

$$a\left(\frac{2\pi}{3}\right) = -8\sin\frac{2\pi}{3} = -8\left(\frac{\sqrt{3}}{2}\right) = -4\sqrt{3} \text{ cm/s}^2$$

THE MASS IS MOVING TO THE LEFT AT $t = \frac{2\pi}{3}$ SECONDS

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(39) $\lim_{x \to 0} \frac{\sin 3x}{x} = \lim_{x \to 0} \frac{3 \sin 3x}{3x}$

$= 3 \lim_{x \to 0} \frac{\sin 3x}{3x}$

$= 3 \lim_{3x \to 0} \frac{\sin 3x}{3x}$, $\theta = 3x$

$= 3 \lim_{\theta \to 0} \frac{\sin \theta}{\theta}$

$= 3(1)$

$= \boxed{3}$

(40) $\lim_{x \to 0} \frac{\sin 4x}{\sin 6x} = \lim_{x \to 0} \frac{\sin 4x}{\sin 6x} \cdot \frac{x}{x} = \lim_{x \to 0} \frac{\sin 4x}{x} \cdot \lim_{x \to 0} \frac{x}{\sin 6x}$

$= \lim_{x \to 0} \frac{4 \sin 4x}{4x} \cdot \lim_{x \to 0} \frac{6x}{6 \sin 6x}$

$= 4 \lim_{x \to 0} \frac{\sin 4x}{4x} \cdot \frac{1}{6} \lim_{x \to 0} \frac{6x}{\sin 6x}$

$= 4 \lim_{4x \to 0} \frac{\sin 4x}{4x} \cdot \frac{1}{6} \lim_{6x \to 0} \frac{6x}{\sin 6x}$

$= 4(1) \cdot \frac{1}{6}(1)$

$= \frac{4}{6}$

$= \boxed{\frac{2}{3}}$