

SECTION 3.3 TRIGONOMETRIC FUNCTIONS

$$\textcircled{1} \quad f(x) = 3x^2 - 2 \cos x$$

$$f'(x) = \frac{d}{dx} 3x^2 - 2 \frac{d}{dx} \cos x$$

$$\boxed{f'(x) = 6x + 2 \sin x}$$

$$\textcircled{2} \quad f(x) = \sqrt{x} \sin x$$

$$u = x^{\frac{1}{2}}$$

$$u' = \frac{1}{2}x^{-\frac{1}{2}}$$

$$f'(x) = \frac{d}{dx} u v$$

$$v = \sin x$$

$$v' = \cos x$$

$$= \left(\frac{1}{2}x^{-\frac{1}{2}}\right)(\sin x) + (x^{\frac{1}{2}})(\cos x)$$

$$\boxed{f'(x) = \frac{\sin x}{2\sqrt{x}} + \sqrt{x} \cos x}$$

$$\textcircled{3} \quad g(t) = 4 \sec t + \tan t$$

$$g'(t) = 4(\sec \tan t) + \sec^2 t$$

$$\boxed{g'(t) = \sec t (4 \tan t + \sec t)}$$

$$\textcircled{4} \quad y = e^u (\cos u + \sin u)$$

$$y = e^u \quad y' = e^u$$

$$y' = \frac{d}{du}(yz)$$

$$z = \cos u + \sin u \quad z' = -\sin u$$

$$y' = e^u(c - \sin u) + e^u(\cos u + \sin u)$$

$$\boxed{y' = e^u(\cos u - \sin u + cu + c)}$$

$$\textcircled{15} \quad f(x) = xe^x \csc x$$

$$u = yz \quad u' = y'z + yz'$$

$$f'(x) = uv$$

$$y = x \quad y' = 1 \quad u = 1(e^x) + x(e^x)$$

$$= u'v + uv'$$

$$z = e^x \quad z' = e^x \quad u' = e^x(1+x)$$

$$= e^x(1+x)(\csc x) + xe^x(-\csc x \cot x) \quad v = \csc x \quad v' = -\csc x \cot x$$

$$= e^x \csc x + xe^x \csc x - xe^x \csc x \cot x$$

$$\boxed{f'(x) = e^x \csc x (1 + x - x \cot x)}$$

$$\textcircled{17} \quad \text{Prove } \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\csc x) = \frac{d}{dx}\left(\frac{1}{\sin x}\right) = \frac{0(\sin x) - (1)(\cos x)}{\sin^2 x} = \frac{-\cos x}{\sin^2 x} = \frac{-1}{\sin x} \cdot \frac{\cos x}{\sin x}$$

$$\boxed{\frac{d}{dx}(\csc x) = -\csc x \cot x}$$

$$\textcircled{18} \quad \text{Prove } \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{(0)(\cos x) - (1)(-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x}$$

$$= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$\boxed{\frac{d}{dx}(\sec x) = \sec x \tan x}$$