SM 121 LESSON 24 1/2/4/5/9/18/26/33/43/47/56 10/3/2008

SECTION 3.2 PRODUCT & QUOTIENT RULES

① $y = (x^2+1)(x^3+1)$

PRODUCT RULE $\frac{d}{dx}(uv) = u'v + uv'$

$y' = (2x)(x^3+1) + (x^2+1)(3x^2)$

$= 2x^4 + 2x + 3x^4 + 3x^2$

$\boxed{y' = 5x^4 + 3x^2 + 2x}$

$y = (x^2+1)(x^3+1)$

$y = x^5 + x^2 + x^3 + 1$

$y = x^5 + x^3 + x^2 + 1$

$\boxed{y' = 5x^4 + 3x^2 + 2x}$

② $F(x) = \frac{x - 3x\sqrt{x}}{\sqrt{x}}$

QUOTIENT RULE $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$

$F'(x) = \frac{\left(1 - \frac{9}{2}x^{\frac{1}{2}}\right)\left(x^{\frac{1}{2}}\right) - \left(x - 3x^{3/2}\right)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{(\sqrt{x})^2}$

$= \frac{x^{\frac{1}{2}} + \frac{9}{2}x - \frac{1}{2}x^{\frac{1}{2}} + \frac{3}{2}x}{x}$

$= \frac{\frac{1}{2}x^{\frac{1}{2}} - 3x}{x}$

$= \frac{1}{2}x^{-\frac{1}{2}} - 3$

$\boxed{F'(x) = \frac{1}{2\sqrt{x}} - 3}$

$F(x) = \frac{x}{x^{\frac{1}{2}}} - \frac{3x\sqrt{x}}{\sqrt{x}}$

$F(x) = x^{\frac{1}{2}} - 3x$

$F'(x) = \frac{1}{2}x^{\frac{3}{2}} - 3$

$\boxed{F'(x) = \frac{1}{2\sqrt{x}} - 3}$

LESSON 24 10/3/2008

④ $g(x) = \sqrt{x}\,e^x$

$g(x) = uv$ $\quad u = \sqrt{x}$ $\quad u' = \frac{1}{2}x^{-\frac{1}{2}}$

$v = e^x$ $\quad v = e^x$

$g'(x) = u'v + uv'$

$= \left(\frac{1}{2}x^{-\frac{1}{2}}\right)\left(e^x\right) + \sqrt{x}\,e^x$

$= \frac{e^x}{2\sqrt{x}} + \sqrt{x}\,e^x$

$= e^x\left(\frac{1}{2\sqrt{x}} - \sqrt{x}\cdot\frac{2\sqrt{x}}{2\sqrt{x}}\right)$

$= e^x\left(\frac{1}{2\sqrt{x}} - \frac{2x}{2\sqrt{x}}\right)$

$= e^x\left(\frac{1-2x}{2\sqrt{x}}\right)$

$$\boxed{g'(x) = \frac{1}{2}e^x x^{-\frac{1}{2}}(1-2x)}$$

⑤ $y = \frac{e^x}{x^2}$ $\quad y = \frac{u}{v}$ $\quad u = e^x$ $\quad u' = e^x$

$v = x^2$ $\quad v' = 2x$

$v^2 = x^4$

$y' = \frac{u'v - uv'}{v^2}$

$= \frac{x^2e^x - (2x)e^x}{x^4}$

$= \frac{xe^x(x-2)}{x^4}$

$$\boxed{y' = \frac{e^x(x-2)}{x^3}}$$

Lesson 24 10/3/2008

(9) $V(x) = (2x^3+3)(x^4-2x)$

$$V'(x) = \left[\frac{d}{dx}(2x^3+3)\right](x^4-2x) + (2x^3+3)\left[\frac{d}{dx}(x^4-2x)\right]$$

$$= (6x^2)(x^4-2x) + (2x^3+3)(4x^3-2)$$

$$= 6x^6 - 12x^3 + 8x^6 - 4x^3 + 12x^3 - 6$$

$$\boxed{V'(x) = 14x^6 - 4x^3 - 6}$$

(18) $y = \dfrac{1}{s+ke^s}$

$u = 1 \qquad u' = 0$

$v = s+ke^s \qquad v' = 1+ke^s$

$v^2 = (s+ke^s)^2$

$$\frac{dy}{ds} = \frac{d}{ds}\left(\frac{u}{v}\right)$$

$$= \frac{u'v - uv'}{v^2}$$

$$= \frac{0(s+ke^s) - (1)(1+ke^s)}{(s+ke^s)^2}$$

$$= \frac{-1-ke^s}{(s+ke^s)^2}$$

$$\boxed{\frac{dy}{ds} = -\frac{1+ke^s}{(s+ke^s)^2}}$$

Lesson 24 10/3/2008

(26) $f(x) = \dfrac{ax+b}{cx+d}$

$u = ax+b \qquad u' = a$

$v = cx+d \qquad v' = c$

$v^2 = (cx+d)^2$

$f'(x) = \dfrac{d}{dx}\left(\dfrac{u}{v}\right)$

$f'(x) = \dfrac{u'v - uv'}{v^2}$

$= \dfrac{a(cx+d) - c(ax+b)}{(cx+d)^2}$

$= \dfrac{\cancel{acx} + ad - \cancel{acx} - bc}{(cx+d)^2}$

$f'(x) = \dfrac{ad-bc}{(cx+d)^2}$

(33) $y = 2xe^x$

$u = 2x \qquad u' = 2$

$v = e^x \qquad v' = e^x$

$y' = \dfrac{d}{dx}uv$

$y' = u'v + uv'$

$y' = 2e^x + 2xe^x$

$y'(0) = 2e^0 + 2(0)e^0$

$y'(0) = 2(1) + 0 = 2$

$m_\perp = \dfrac{-1}{m} = \dfrac{-1}{2}$

TANGENT $m = 2 \quad (0,0)$

$y - y_1 = m(x - x_1)$

$y - 0 = 2(x - 0)$

$\boxed{y = 2x \quad \text{TANGENT LINE}}$

NORMAL LINE $m = -\frac{1}{2} \quad (0,0)$

$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{1}{2}(x - 0)$

$\boxed{y = -\frac{1}{2}x \quad \text{NORMAL LINE}}$

Lesson 24 10/3/2008

43 (a) $(fg)'(5) = f'(5)g(5) + f(5)g'(5)$

$f(5) = 1$ $g(5) = -3$

$f'(5) = 6$ $g'(5) = 2$

$= (6)(-3) + (1)(2)$

$= -18 + 2$

(a) $= \boxed{-16}$

(b) $\left(\frac{f}{g}\right)'(5) = \dfrac{f'(5)g(5) - f(5)g'(5)}{[g(5)]^2}$

$= \dfrac{(6)(-3) - (1)(2)}{(-3)^2}$

$= \dfrac{-18 - 2}{9}$

(b) $= \boxed{\dfrac{-20}{9}}$

(c) $\left(\frac{g}{f}\right)'(5) = \dfrac{g'(5)f(5) - g(5)f'(5)}{[f(5)]^2}$

$= \dfrac{(2)(1) - (-3)(6)}{(1)^2}$

$= \dfrac{2 + 18}{1}$

$= \boxed{20}$

LESSON 24 10/3/2008

47 a $u = f(x)g(x)$

$$u'(1) = f'(1)g(1) + f(1)g'(1)$$

$$= (2)(1) + (2)(-1)$$

$$= 2 - 2$$

a $\boxed{u'(1) = 0}$

b $v = \dfrac{f(x)}{g(x)}$

$$v'(5) = \frac{f'(5)g(5) - f(5)g'(5)}{[g(5)]^2}$$

$$= \frac{(-\frac{1}{3})(2) - (\frac{2}{3})(3)}{(2)^2}$$

$$= \frac{-\frac{2}{3} - 2}{4}$$

$$= \frac{-\frac{8}{3}}{4}$$

$$= -\frac{\cancel{8}^{2}}{3} \cdot \frac{1}{\cancel{4}_{1}}$$

$$\boxed{v'(5) = \frac{-2}{3}}$$

LESSON 24 10/3/2008

(56) $F(x) = f(x)g(x)$ SHOW $F''(x) = f''g + 2f'g' + fg''$

$F(x) = uv$ $u = f$ $u' = f'$

$v = g$ $v' = g'$

$F'(x) = f'g + fg'$

$F''(x) = uv + yz$ $u = f'$ $u' = f''$

$v = g$ $v' = g'$

$= u'v + uv' + y'z + yz'$ $y = f$ $y' = f'$

$F''(x) = f''g + f'g' + f'g' + fg''$ $z = g'$ $z' = g''$

(a)

$$\boxed{F''(x) = f''g + 2f'g' + fg''}$$

$r = f''$ $r' = f'''$

(b) $F''' = r's + rs' + 2(u'v + uv') + y'z + yz'$ $s = g$ $s' = g''$

$F''' = f'''g + f''g' + 2(f''g' + f'g'') + f'g'' + fg'''$ $u = f'$ $u' = f''$

$v = g'$ $v' = g''$

(b)

$$\boxed{F''' = f'''g + 3f''g' + 3f'g'' + fg'''}$$

$y = f$ $y' = f'$

$z = g''$ $z = g'''$

(b)

$$\boxed{F^{(4)} = f^{(4)}g + 4f'''g'' + 6f''g'' + 4f'g''' + fg^{(4)}}$$

(c) BY THE BINOMIAL THEOREM

$$F^{(n)} = f^{(n)}g + nf^{(n-1)}g' + \binom{n}{2}f^{(n-2)}g'' + \cdots + \binom{n}{k}f^{(n-k)}g^{n} + \cdots + nf'g^{(n-1)} + fg^{(n)}$$

WHERE $\binom{n}{k} = \frac{n!}{k!(n-k)!}$