SM121 LESSON 18 1/3/6/11/13/17/18/19 9/22/2008

SECTION 2.7 DERIVATIVES & RATES OF CHANGE

① a) SLOPE OF SECANT LINE THROUGH $P(3, f(3))$ AND $Q(x, f(x))$

$$\boxed{m_{PQ} = \frac{f(x) - f(3)}{x - 3}}$$

b) SLOPE OF TANGENT AT P

$$\boxed{m = \lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}}$$

③ a i)

$$m = \lim_{x \to 1} \frac{f(x) - 3}{x - 1} = \lim_{x \to 1} \frac{4x - x^2 - 3}{x - 1} = \lim_{x \to 1} \frac{-(x^2 - 4x + 3)}{x - 1}$$

$$= \lim_{x \to 1} \frac{-\cancel{(x-1)}(x-3)}{\cancel{(x-1)}}$$

$$= \lim_{x \to 1} -(x - 3)$$

$$= -(1 - 3)$$

$$= -(-2)$$

$$= \boxed{2}$$

ii)

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{4(1+h) - (1+h)^2 - 3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{4} + 4h - (\cancel{1} + 2h + h^2) - \cancel{3}}{h}$$

$$= \lim_{h \to 0} \frac{-h^2 + 2h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2 - h)}{\cancel{h}}$$

$$= \lim 2 - \cancel{h}^{\,0} = \boxed{2}$$

LESSON 18 9/22/2008

③ b) $m = 2 \quad (1, 3)$

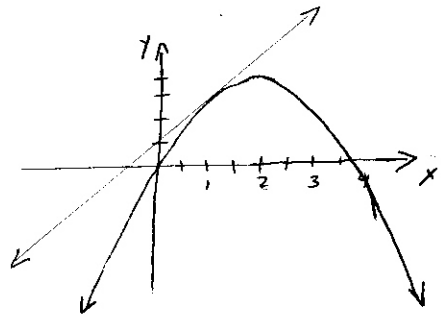
$y - y_1 = m(x - x_1)$

$y - 3 = 2(x - 1)$

$y - 3 = 2x - 2$

$\boxed{y = 2x + 1}$

c) USING CALCULATOR ...



⑥ $y = 2x^3 - 5x$ FIND TANGENT LINE AT $(-1, 3)$

$$m = \lim_{h \to 0} \frac{f(-1+h) + f(-1)}{h} = \lim_{h \to 0} \frac{2(-1+h)^3 - 5(-1+h) - 3}{h}$$

$$= \lim_{h \to 0} \frac{2(-1+h)(-1+h)^2 + 5 - 5h - 3}{h}$$

$$= \lim_{h \to 0} \frac{2(-1+h)(1-2h+h^2) - 5h + 2}{h}$$

$$= \lim_{h \to 0} \frac{2(-1+2h-h^2+h-2h^2+h^3) - 5h + 2}{h}$$

$$= \lim_{h \to 0} \frac{-2+4h-2h^2+2h-4h^2+2h^3-5h+2}{h}$$

$$= \lim_{h \to 0} \frac{2h^3 - 6h^2 + h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2h^2 - 6h + 1)}{\cancel{h}}$$

$y - y_1 = m(x - x_1)$

$y - 3 = 1(x - (-1))$

$y - 3 = x + 1$

$\boxed{y = x + 4}$

EQUATION FOR TANGENT LINE AT $(-1, 3)$

LESSON 18 9/22/2008

(11) a) THE PARTICLE IS MOVING RIGHT FROM $t>0$ TO $t<1$ AND $t>4$ TO $t<6$

$$(0,1) \cup (4,6)$$

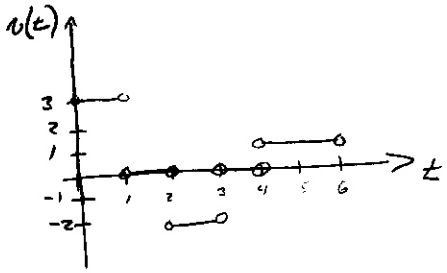
THE PARTICLE IS MOVING LEFT FROM $t>2$ TO $t<3$

$$(2,3)$$

THE PARTICLE IS STILL FROM $t>1$ TO $t<2$ AND $t>3$ TO $t<4$

$$(1,2) \cup (3,4)$$

b) DRAW A GRAPH OF VELOCITY



(13) $s(t) = 40t - 16t^2$ FIND $v(2)$

$$s(2) = 40(2) - 16(2)^2$$
$$= 80 - 16(4)$$
$$= 80 - 64$$
$$s(t) = 16$$

$$v(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$$

$$v(2) = \lim_{h \to 0} \frac{s(2+h) - 16}{h}$$

$$= \lim_{h \to 0} \frac{40(2+h) - 16(2+h)^2 - 16}{h}$$

$$= \lim_{h \to 0} \frac{80 + 40h - 16(4 + 4h + h^2) - 16}{h}$$

$$= \lim_{h \to 0} \frac{80 + 40h - 64 - 64h - 16h^2 - 16}{h}$$

$$= \lim_{h \to 0} \frac{-16h^2 - 24h}{h} = \lim_{h \to 0} \frac{\cancel{h}(-16h - 24)}{\cancel{h}}$$

$$\lim_{h \to 0} -16h - 24 = \boxed{-24 \text{ ft/s}}$$

LESSON 18 9/22/2008

(17) 0 $\quad g'(2) \approx 1$

$g'(-2) \approx 2$

$g'(0) \approx -1$ $\quad g'(4) \approx \frac{1}{2}$

$$\boxed{g'(0) < 0 < g'(4) < g'(2) < g'(-2)}$$

(18) a) $g'(5) = 4 \quad (5, -3)$

$$y - y_1 = m(x - x_1)$$

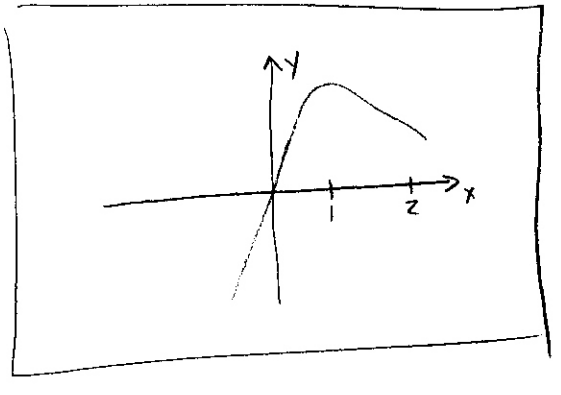
$$y - (-3) = 4(x - 5)$$

$$y + 3 = 4x - 20$$

a) $\boxed{y = 4x - 23}$

b) $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 3}{0 - 4} = \frac{-1}{-4} = \frac{1}{4}$

$$\boxed{\begin{aligned} f'(4) &= \tfrac{1}{4} \\ f(4) &= 3 \end{aligned}}$$

(19) $f(0) = 0 \quad f'(1) = 0$

$f'(0) = 3 \quad f'(2) = -1$