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SECTION 2.6 LIMITS INVOLVING INFINITY

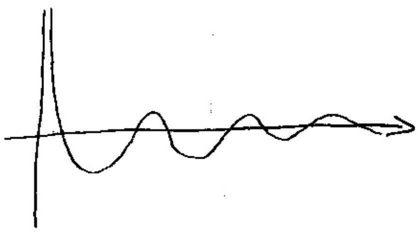
① a) $\lim_{x \to \infty} f(x) = 5$

AS THE FUNCTION f GOES TO INFINITY THE VALUE OF $f(x)$ APPROACHES 5. THERE IS A HORIZONTAL ASYMPTOTE AT $y = 5$

b) $\lim_{x \to -\infty} f(x) = 3$

AS THE FUNCTION f GOES TO NEGATIVE INFINITY THE VALUE OF $f(x)$ APPROACHES 3. THERE IS A HORIZONTAL ASYMPTOTE AT $y = 3$.

② a) A GRAPH CANNOT INTERCEPT A VERTICAL ASYMPTOTE, BUT IT CAN INTERCEPT A HORIZONTAL ASYMPTOTE.

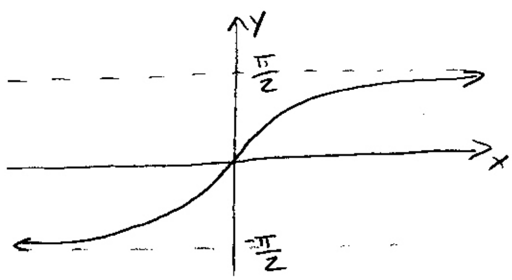


VERT. ASYMPTOTE $x = 0$

HORIZ. ASYMPTOTE $y = 0$

b) A GRAPH CAN HAVE <u>AT MOST</u> TWO HORIZONTAL ASYMPTOTES.

EXAMPLE: $f(x) = \arctan x$



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(4)

a) $\lim_{x\to\infty} g(x) = \boxed{2}$

b) $\lim_{x\to-\infty} g(x) = \boxed{-2}$

c) $\lim_{x\to 3} g(x) = \boxed{\infty}$

d) $\lim_{x\to 0} g(x) = \boxed{-\infty}$

e) $\lim_{x\to -2^+} g(x) = \boxed{-\infty}$

f) ASYMPTOTES

$x = -2$

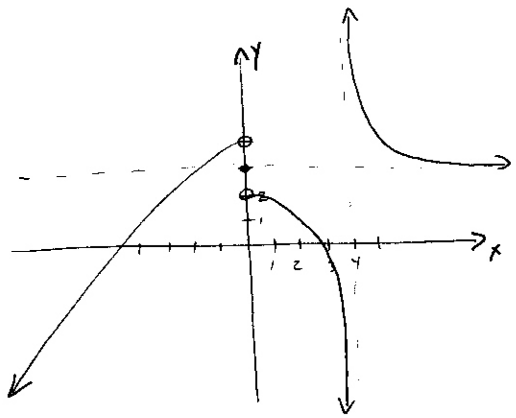
$x = 0$

$x = 3$

$y = -2$

$y = 2$

(9)



$f(0) = 3$

$\lim_{x\to 0^-} f(x) = 4$

$\lim_{x\to 0^+} f(x) = 2$

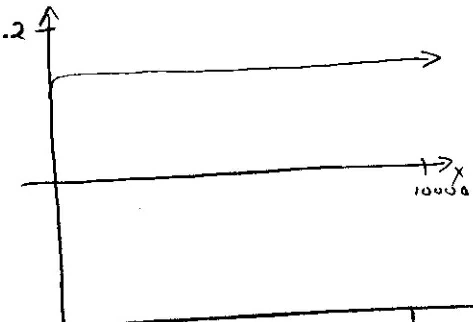
$\lim_{x\to -\infty} f(x) = -\infty$

$\lim_{x\to 4^-} f(x) = -\infty$

$\lim_{x\to 4^+} f(x) = \infty$

$\lim_{x\to \infty} f(x) = 3$

(12)



USING A CALCULATOR:

$\lim_{x\to\infty}\left(1 - \frac{2}{x}\right)^x \approx 0.14$

b)

x	f(x)
10000	0.13531
100000	0.13533
1000000	0.13534
10^{10}	0.13534

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(17) $\lim\limits_{x \to -\infty} \dfrac{1 - x - x^2}{2x^2 - 7} \cdot \dfrac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$\lim\limits_{x \to -\infty} \dfrac{\frac{1}{x^2} - \frac{1}{x} - 1}{2 - \frac{7}{x^2}}$

$\dfrac{\cancel{\frac{1}{(-\infty)^2}} \to 0 - \cancel{\frac{1}{(-\infty)}} \to 0 - 1}{2 - \cancel{\frac{7}{(-\infty)^2}} \to 0}$

$\dfrac{0 - 0 - 1}{2 - 0}$

$\boxed{-\frac{1}{2}}$

(20) $\lim\limits_{t \to -\infty} \dfrac{t^2 + 2}{t^3 + t^2 - 1} \cdot \dfrac{\frac{1}{t^3}}{\frac{1}{t^3}} = \lim\limits_{t \to -\infty} \dfrac{\frac{1}{t} + \frac{2}{t^3}}{1 + \frac{1}{t} - \frac{1}{t^3}} = \dfrac{\cancel{\frac{1}{(-\infty)}} \to 0 + \cancel{\frac{2}{(-\infty)}} \to 0}{1 + \cancel{\frac{1}{(-\infty)}} \to 0 - \cancel{\frac{1}{(-\infty)^3}} \to 0} = \dfrac{0}{1 + 0 + 0} = \boxed{0}$

(42) $y = \dfrac{1 + x^4}{x^2 - x^4}$

VERTICAL ASYMPTOTES

$x^2 - x^4 = 0$

$x^2(1 - x^2) = 0$

$x^2(1 - x)(1 + x) = 0$

$\boxed{x = -1, \; x = 1, \; x = 0 \text{ VERTICAL ASYMPTOTES}}$

HORIZONTAL ASYMPTOTES

$\lim\limits_{x \to -\infty} \dfrac{1 + x^4}{x^2 - x^4} \cdot \dfrac{\frac{1}{x^4}}{\frac{1}{x^4}}$

$\lim\limits_{x \to -\infty} \dfrac{\frac{1}{x^4} + 1}{\frac{1}{x^2} - 1} = \dfrac{0 + 1}{0 - 1} = \boxed{-1}$

$\lim\limits_{x \to \infty} \dfrac{1 + x^4}{x^2 - x^4} \cdot \dfrac{\frac{1}{x^4}}{\frac{1}{x^4}}$

$\lim\limits_{x \to \infty} \dfrac{\frac{1}{x^4} + 1}{\frac{1}{x^2} - 1} = \dfrac{0 + 1}{0 - 1} = \boxed{-1}$

y = -1 HORIZONTAL

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(55) $\lim_{x\to\infty} \dfrac{P(x)}{Q(x)}$

a) DEGREE OF $P<Q$: NUMERATOR $\to 0$, DENOMINATOR DOESN'T $\therefore \boxed{\lim_{x\to\infty} \dfrac{P(x)}{Q(x)} = 0}$

b) DEGREE OF $P>Q$: NUMERATOR $\to \pm\infty$, DENOMINATOR DOESN'T $\therefore \boxed{\lim_{x\to\infty} \dfrac{P(x)}{Q(x)} = \pm\infty}$

(57) USE SQUEEZE THEOREM:

$$\lim_{x\to\infty} \frac{10e^x - 21}{2e^x} < \lim_{x\to\infty} f(x) < \lim_{x\to\infty} \frac{5\sqrt{x}}{\sqrt{x-1}}$$

$$\lim_{x\to\infty} \frac{10e^x - 21}{2e^x} \cdot \frac{\frac{1}{e^x}}{\frac{1}{e^x}} < \lim_{x\to\infty} f(x) < \lim_{x\to\infty} \frac{5\sqrt{x}}{\sqrt{x-1}} \cdot \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}}$$

$$\lim_{x\to\infty} \frac{\frac{10e^x}{e^x} - \frac{21}{e^x}}{\frac{2e^x}{e^x}} < \lim_{x\to\infty} f(x) < \lim_{x\to\infty} \frac{\frac{5\sqrt{x}}{\sqrt{x}}}{\sqrt{\frac{x}{x} - \frac{1}{x}}}$$

$$\lim_{x\to\infty} \frac{10 - \frac{21}{e^x}}{2} < \lim_{x\to\infty} f(x) < \lim_{x\to\infty} \frac{5}{\sqrt{1-\frac{1}{x}}}$$

$$\frac{10 - \cancel{\frac{21}{e^\infty}}^{0}}{2} < \lim_{x\to\infty} f(x) < \frac{5}{\sqrt{1-\cancel{\frac{1}{\infty}}^{0}}}$$

$$5 < \lim_{x\to\infty} f(x) < 5$$

$$\boxed{\therefore \lim_{x\to\infty} f(x) = 5}$$