Forward Kinematics

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- Kinematics is the relationships between the positions, velocities and accelerations of the links of a manipulator.
- The problem we're eventually trying to solve is: we know where we want our robot is, what position are the motors in? This in Inverse Kinematics, and is important because we can use the same technique to solve: we know what position we *want* to be in, what position would the motors be in? Once we know that, we can compare where the motors are now to where the motors should be, and make them so.
- To understand that, we first need to understand the problem of: We know what position the motors are in, where is the arm? This is know as the forward kinematics problem.
  - We will focus on the positions.
  - The velocities and accelerations are based on the first and second derivatives of the positions, naturally, but we won't discuss them in this class.
- Serial link manipulator is a series of links which connects the hand of the robot to the base.
  - Each link is connected to the next by an actuated joint (i.e. one that the robot can move).
  - The relationship between neighboring links can be described with (yet another) homogeneous transformation matrix, denoted **A**.
  - We use a series of A matrices to describe the transform from the base of the robot to the hand of the manipulator- This is called the forward kinematic transform.
- This transform is part of a closed equation involving the position transform from the previous discussion on space:

$$^{R}T_{H} = {}^{R}T_{L_1}\,{}^{L_1}T_{L_2}\,{}^{L_2}T_{L_3}\cdots{}^{L_{n-1}}T_{L_n}$$

  we'll abbreviate each ${}^{L_{m-1}}T_{L_m}$ as A_m.
- In order to calculate the hand position, we must:
  - Set up coordinate frames for each link in the hand,

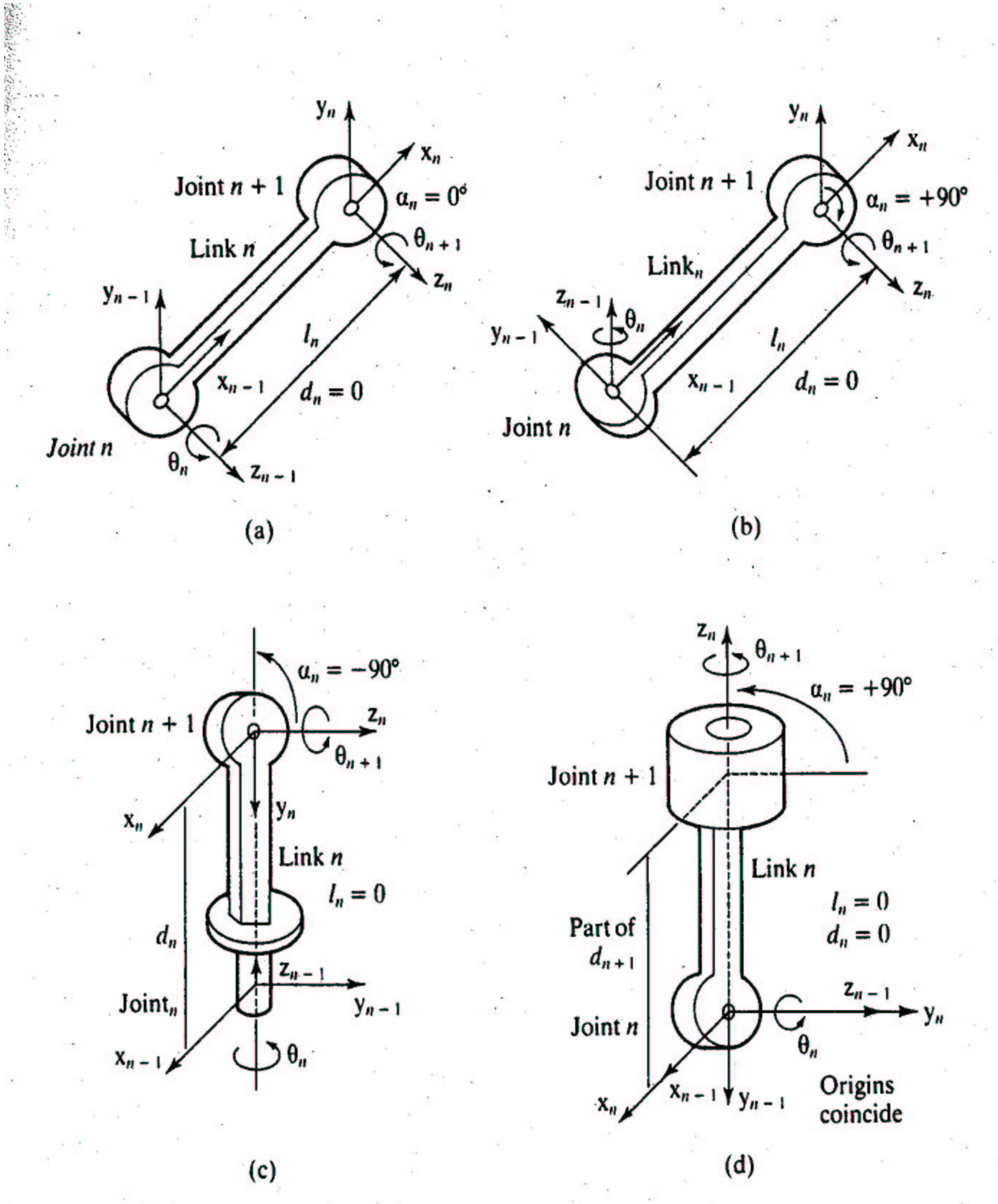
- Generate **A** matrices for each transformation between the frames,
- Multiply the **A** matrices to generate the right hand side of the above equation,
- Solve the equation for the various pieces we need: the position of the hand frame, as well as its orientation.

- The method of setting up coordinated frames is somewhat arbitrary, but we'll use one standard (Paul, 1981).
- Setup:
  - Define: Link- a rigid piece of the robot arm that connects two joints. It therefore maintains a fixed relationship between the joints at its ends.
  - Define: Joint- a connection between two links which allows the links to either rotate of translate w/r/t each other. When they rotate, it is called a revolute joint, when they translate, its called a prismatic- no idea why. We will mostly discuss revolute joints cause that's what we have in our robots. But note that the fingers are prismatic.
  - Define: Base- a link that does not move w/r/t the robot frame.
  - Define: Proximal- of two things (links, joints etc.) the one closer to the robot base in the chain of links.
  - Define: Distal - of two things (links, joints etc.) the one farther from the robot base in the chain of links.
  - Define: Joint Axis- the axis around which the revolute joint turns.
  - Start at the base, number the links from 0 to n. The base is 0.
  - Number the joints from 1 to n+1. The endpoint of the robot arm is n+1.
  - Base frame will be located at joint 1. Why? Why not the bottom of the base?
  - z-axis of the base frame is the joint axis of joint 1.
  - x-axis of the base frame should be in the direction you want to make the start position of link 1.
- Repeatedly assign coordinate frame to each link:
  - To locate the origin of the frame:
    * identify the 2 joint axes of the joints at either end of the link.
    * If the joint axes do not intersect:
      · There should be a line which passes through the origin of the previous coordinate frame that is perpendicular to both joint axes.
      · That line is called the common normal to the two joint axes.
      · place the origin of the frame at the intersection of the common normal and the distal joint axis.

        * if the joint axes do intersect, then locate the origin at the intersection of the joint axes.

    - Make the z axis the distal joint axis. What direction should the z be? It doesn't really matter, but its best if we're consistent as a class. We'll start with z pointing up, and talk about other cases later.
    - If the 2 joint axes do not intersect, then make the x axis coincident with the the common normal to the 2 joint axes of the link. Make x point away from the proximal joint.
    - If the joint axes intersect, then make the x axis perpendicular to the plane defined by the 2 joint axes.
    - Make the y axis correct using the right hand rule.
    - At the hand/gripper set a coordinate frame at the center of the gripper area (the grip location)
        * make the z-axis parallel to the z-axis of the previous coordinate frame.
        * make the z-y plane parallel to the plane of the hand.
        * Set x-axis so that is is parallel and in the same direction as the previous x-axis.

- See some examples.



- Look at the examples above and note that the *relation* between two adjacent coordinate frames can be described by the following parameters:

- l_n - the length parameter. The distance between the two z axes. Note that if the z axes intersect, then $l_n = 0$.
- d_n - the distance parameter. The distance between the the two x axes. Often this too can be 0.
- θ_n- the link angle. The angle between link n-1 and link n.
- α_n- the twist. The angle of twist between the two joints in the link itself.

- We define the A matrices using these parameters:
  - $A_n = Rot(z, \theta)Trans(0, 0, d)Trans(l, 0, 0)Rot(x, \alpha)$.

$$\begin{bmatrix} \cos(\Theta) & -\sin(\Theta) & 0 & 0 \\ \sin(\Theta) & \cos(\Theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) & 0 \\ 0 & \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\Theta) & -\sin(\Theta)\cos(\alpha) & \sin(\Theta)\sin(\alpha) & l\cos(\Theta) \\ \sin(\Theta) & \cos(\Theta)\cos(\alpha) & -\cos(\Theta)\sin(\alpha) & l\sin(\Theta) \\ 0 & \sin(\alpha) & \cos(\alpha) & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - Finally, we multiply the A matrices together to get the transformation T: $T = A_1 A_2 A_3 ... A_n$

- Next the question becomes, given a T calculated by multiplying the A matrices, can we determine the position and orientation of the end of the arm?
  - given a T:

$$\begin{bmatrix} x_x & y_x & z_x & p_x \\ x_y & y_y & z_y & p_y \\ x_z & y_z & z_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

    calculated by the product of A's,
  - recall that T is also the product of translation and rotation from the reference frame:

    $T = Trans(x, y, z)Rot(z, \phi)Rot(y, \beta)Rot(x, \psi)$.

    which is:

$$\begin{bmatrix} \cos(\phi)\cos(\beta) & \cos(\phi)\sin(\beta)\sin(\psi) - \sin(\phi)\cos(\psi) & \cos(\phi)\sin(\beta)\cos(\psi) + \sin(\phi)\sin(\psi) & p_x \\ \sin(\phi)\cos(\beta) & \sin(\phi)\sin(\beta)\sin(\psi) + \cos(\phi)\cos(\psi) & \sin(\phi)\sin(\beta)\cos(\psi) - \cos(\phi)\sin(\psi) & p_y \\ -\sin(\beta) & \cos(\beta)\sin(\psi) & \cos(\beta)\cos(\psi) & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We set our 2 versions of T equal to each other and solve for the position p as well as ϕ, β, and ψ.

$$\beta = -\arcsin(x_z)$$
$$\psi = \arcsin(\frac{y_z}{\cos(\beta)})$$
$$\phi = \arcsin(\frac{x_y}{\cos(\beta)})$$

– And p is just the rightmost column.